Web-based Supplementary Materials for

Marginal Regression Analysis of Recurrent Events with Coarsened Censoring Times

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Web Appendix A: Asymptotics Derivation

We establish consistency and weak convergence of the estimators derived in Section 3 of the paper under the following assumptions, which are sufficient but not necessarily necessary conditions. The notation introduced in Sections 2 and 3 is used below.

(i) $\{N_i(\cdot), Z_i, B_i\}(i = 1, \ldots, n)$ are independent and identically distributed;

(ii) $P(C_{Li} < \tau_L) > 0, P(C_{Ri} > \tau_R) > 0 (i = 1, \ldots, n)$, where $0 < \tau_L < \tau_R < 18$ are predetermined constants;

(iii) $N_i(18)(i = 1, \ldots)$ are bounded by a constant;

(iv) Over $(0, 18)$, $\lambda_0(\cdot) > 0$ and continuous and all the components of $\beta(\cdot)$ have a continuous second derivative;

(v) The kernel function $K(\cdot)$ is a bounded and symmetric density with a bounded support;

(vi) The bandwidth $h \to 0$ and $h = O(n^{-\nu})$ with $1/2 < \nu < 1$ as $n \to \infty$;

(vii) The matrix $\{v^{(2)}(\beta_0; a) - v^{(1)}(\beta_0; a)^{\otimes 2}/v^{(0)}(\beta_0; a)\}$ with $v^{(q)}(\beta; a) = E\{Y(a|B_1) Z_1^{\otimes q} e^{\beta(a)' Z_1}\}$ for $q = 0, 1, 2$, is positive definite for $a \in (0, 18)$.

A.1 Asymptotics of $\hat{\beta}_n(\cdot|\mathbf{B})$

We establish in the following the pointwise consistency and weak convergence of the estimator $\hat{\beta}_n(\cdot|\mathbf{B})$ derived in Section 3.2.1, and present a consistent estimator for its asymptotic variance. The proof below utilizes approaches similar to those of Bilias et al. (1997), Lin et al. (2000) and Hu et al. (2003). We denote the true value of $\beta(a)$ by $\beta_0(a)$; $\gamma = \left(\beta(a)', \dot{\beta}(a)'\right)'$, by γ_0.

A.1.1 Consistency of $\hat{\beta}_n(a|\mathbf{B})$.

Let $dM_i(u; \beta_0(u)) = Y(u|B_i)\left[dN_i(u) - \exp\{\beta_0(u)' Z_i\} d\Lambda_0(u)\right]$ for $u \in (0, 18)$. Thus the expectation $E\{dM_i(u; \beta_0(u))|Z_i, B_i\} = 0$ for $u \in (0, 18)$. The estimating function $U_n(\gamma; a|\mathbf{B})$ in (6) is

$$\frac{1}{n}\int_0^{18} K_h(u-a) \sum_{i=1}^n \{Z_i^*(u,a) - \bar{Z}_n^*(\gamma; u, a)\} \left[dM_i(u; \beta_0(u)) + Y(u|B_i)\exp\{\beta_0(u)' Z_i\} d\Lambda_0(u)\right].$$

We can show both $\{\{M_i(u;\beta_0), u \in (0,18)\}: \ i = 1,\ldots,n\}$ and $\{\{\int_0^u Z_i^*(t,a)dM_i(t;\beta_0), u \in (0,18)\}: \ i = 1,\ldots,n\}$ are manageable (Pollard, 1990). Hence the sequence of $\Big\{(n^{-1/2}\sum_{i=1}^n M_i(u;\beta_0),\, n^{-1/2}\sum_{i=1}^n \int_0^u Z_i^*(t,a)dM_i(t;\beta_0))'\Big\}$ converges weakly to a zero-mean Gaussian process with index $u \in (0,18)$, say $(W_M, W_{M_z})'$. By the strong law of large numbers, $S_n^{(q)}(\gamma;u,a)$ converges a.s. to $s^{(q)}(\gamma;u,a) = E\Big[Y(u|B_1)Z_1^*(u,a)^{\otimes q}\exp\{\gamma' Z_1^*(u,a)\}\Big]$ for $q = 0,1,2$, and thus $\bar{Z}_n^*(\gamma;u,a)$ to $\bar{z}^*(\gamma;u,a) = s^{(1)}(\gamma;u,a)/s^{(0)}(\gamma;u,a)$ and $S_n^{(2)}(\gamma;u,a)/S_n^{(0)}(\gamma;u,a)$ to $s^{(2)}(\gamma;u,a)/s^{(0)}(\gamma;u,a)$. Note further that, as $n \to \infty$ and with fixed a, $\int_0^{18} K_h(u-a)dH_n(u,a) \to h(a,a)$ if $\partial H_n(u,a)/\partial u \to h(u,a)$ a.s. We can show that $\tilde{U}_n(\gamma_0;a|\mathbf{B})$ converges a.s. to 0 and so does $U_n(\gamma_0;a|\mathbf{B})$.

Note that

$$\frac{\partial U_n(\gamma;a|\mathbf{B})}{\partial\gamma} = -\frac{1}{n}\int_0^{18} K_h(u-a)\left[\frac{S_n^{(2)}(\gamma;u,a)}{S_n^{(0)}(\gamma;u,a)} - \left\{\frac{S_n^{(1)}(\gamma;u,a)}{S_n^{(0)}(\gamma;u,a)}\right\}^{\otimes 2}\right]\sum_{i=1}^n Y(u|B_i)dN_i(u),$$

and $\partial U_n(\gamma_0;a|\mathbf{B})/\partial\gamma$ converges a.s. to the non-positive definite matrix

$$-\left\{s^{(2)}(\gamma_0;a,a) - \frac{s^{(1)}(\gamma_0;a,a)^{\otimes 2}}{s^{(0)}(\gamma_0;a,a)}\right\}\lambda_0(a) = -\begin{pmatrix} \Pi(\beta_0;a) & 0 \\ 0 & 0 \end{pmatrix},$$

of which the block matrix $\Pi(\beta_0;a) = \{v^{(2)}(\beta_0;a) - v^{(1)}(\beta_0;a)^{\otimes 2}/v^{(0)}(\beta_0;a)\}\lambda_0(a)$ is positive definite by Conditions *(iv)* and *(vii)*.

By the Taylor expansion of $U_n(\gamma;a|\mathbf{B})$ about point γ_0 for $\gamma \in \mathcal{B}_\epsilon = \{\gamma : ||\gamma-\gamma_0|| \le \epsilon\}$ with $\epsilon > 0$, we have

$$U_n(\gamma;a|\mathbf{B}) = U_n(\gamma_0;a|\mathbf{B}) + \frac{\partial U_n(\gamma_0;a|\mathbf{B})}{\partial\gamma}(\gamma-\gamma_0) + o\big((\gamma-\gamma_0)\big). \tag{1}$$

We choose ϵ to be small enough such that the upper left block matrix of $\partial U_n(\gamma;a|\mathbf{B})/\partial\gamma$ is negative definite with a correspondingly large n, and thus the first component vectors of the solutions to $U_n(\gamma;a|\mathbf{B}) = 0$ in $\mathcal{B}_\epsilon$ must be the same. Let $\gamma_1 = \gamma_0 - \epsilon\mathbf{1}$ and $\gamma_2 = \gamma_0 + \epsilon\mathbf{1}$, where $\mathbf{1}$ is the vector with all the components the same and $||\mathbf{1}|| = 1$. If n is sufficiently large, the interval with the two endpoints $\min(U_n(\gamma_1;a|\mathbf{B}), U_n(\gamma_2;a|\mathbf{B}))$ and $\max(U_n(\gamma_1;a|\mathbf{B}), U_n(\gamma_2;a|\mathbf{B}))$ contains a solution of $U_n(\gamma;a|\mathbf{B}) = 0$. That is, $U_n(\gamma;a|\mathbf{B}) = 0$ has solutions in $\mathcal{B}_\epsilon$ with the same first component vector, which is $\hat{\beta}_n(a|\mathbf{B})$. This proves the consistency of $\hat{\beta}_n(a|\mathbf{B})$ since ϵ can be arbitrarily small.

A.1.2 Weak convergence of $\hat{\beta}_n(a|\mathbf{B})$: $\sqrt{nh}(\hat{\beta}_n(a|\mathbf{B}) - \beta_0(a)) \xrightarrow{d} N(0, AV(\beta_0;a))$.

By the strong embedding theorem (cf. Shorack and Wellner (1986), Theorem 2.3.4), we can construct a new probability space and have the sequence $\Big\{\big(n^{-1/2}\sum_{i=1}^n M_i(u;\beta_0), n^{-1/2}\sum_{i=1}^n\int_0^u Z_i^*(t,a)dM_i(t;\beta_0), S_n^{(0)}(\gamma_0;u,a), S_n^{(1)}(\gamma_0;u,a)\big) : u \in (0,18)\Big\}$ a.s. convergence to $\{W_M, W_{M_z}, s^{(0)}(\gamma_0;\cdot,a), s^{(1)}(\gamma_0;\cdot,a)\}$. Then we may show that a.s., as $n \to \infty$ for $u \in (0,18)$, $n^{-1/2}\sum_{i=1}^n\int_0^u\{\bar{z}^*(\gamma;t,a) - \bar{Z}_n^*(\gamma;t,a)\}dM_i(t,\beta_0) \to 0$, and $\sqrt{nh}U_n(\gamma_0;a|\mathbf{B})$ is asymptotically normal in the original space with mean 0.

By the Taylor expansion in (1) and the consistency of $\hat{\gamma}_n$ with γ_0, we see that $\sqrt{nh}(\hat{\gamma}_n - \gamma_0)$ is asymptotically equivalent to $\{-\partial U_n(\gamma_0;a|\mathbf{B})/\partial\gamma\}^-\{\sqrt{nh}U_n(\gamma_0;a|\mathbf{B})\}$. This yields the asymptotical normality of $\sqrt{nh}(\hat{\beta}_n(a|\mathbf{B}) - \beta_0(a))$, the first component vector of $\sqrt{nh}(\hat{\gamma}_n - \gamma_0)$, with mean zero and variance matrix $AV(\beta_0;a) = \Pi(\beta_0;a)^{-1}\Sigma(\beta_0;a)\Pi(\beta_0;a)^{-1}$, where $\Sigma(\beta_0;a)$ is the asymptotic variance of the first component vector of $\sqrt{nh}U_n(\gamma_0;a|\mathbf{B})$.

A.1.3 Consistent estimator of $AV(\beta_0;a)$.

The upper left block of $-\partial U_n(\gamma_0;a|\mathbf{B})/\partial\gamma$ is

$$\hat{\Pi}(\gamma;a|\mathbf{B}) = \frac{1}{n}\int_0^{18} K_h(u-a)\left[\frac{S_{1n}^{(2)}(\gamma;u,a)}{S_{1n}^{(0)}(\gamma;u,a)} - \left\{\frac{S_{1n}^{(1)}(\gamma;u,a)}{S_{1n}^{(0)}(\gamma;u,a)}\right\}^{\otimes 2}\right]\sum_{i=1}^{n} dN_i^*(u)$$

with $S_{1n}^{(q)}(\gamma;u,a) = \sum_{i=1}^n Y(u|B_i)Z_i^{\otimes q}\exp\{\gamma' Z_i^*(u,a)\}/n$ for $q=0,1,2$. Thus $\hat{\Pi}(\hat{\gamma}_n;a|\mathbf{B})$ is a consistent estimator of $\Pi(\beta_0;a)$.

Note that the first component vector of $\sqrt{nh}\tilde{U}_n(\gamma;a|\mathbf{B})$ is $n^{-1/2}\sum_{i=1}^n \tilde{X}_i(\gamma;a)$ with independent $\tilde{X}_i(\gamma;a) = \int_0^{18}\sqrt{h}K_h(u-a)\{Z_i - s_1^{(1)}(\gamma;u,a)/s^{(0)}(\gamma;u,a)\}dN_i^*(u)$. So $\Sigma(\beta_0;a)$, the limit of the variance of $\sqrt{nh}U_n(\gamma;a|\mathbf{B})$'s first component vector, is consistently estimated by $\hat{\Sigma}(\hat{\gamma}_n;a|\mathbf{B}) = \sum_{i=1}^n \big(X_i(\hat{\gamma}_n;a) - \bar{X}(\hat{\gamma}_n;a)\big)^{\otimes 2}/(n-1)$ with $X_i(\gamma;a) = \int_0^{18}\sqrt{h}K_h(u-a)\big[Z_i - S_{1n}^{(1)}(\gamma;u,a)/S_n^{(0)}(\gamma;u,a)\big]dN_i^*(u)$ and $\bar{X}(\gamma;a) = \sum_{i=1}^n X_i(\gamma;a)/n$.

Thus we obtain a consistent estimator for $AV(\beta_0;a)$): $\hat{\Pi}(\hat{\gamma}_n;a|\mathbf{B})^{-1}\hat{\Sigma}(\hat{\gamma}_n;a|\mathbf{B})\hat{\Pi}(\hat{\gamma}_n;a|\mathbf{B})^{-1}$.

A.2 Asymptotics of $\hat{\beta}_n(\cdot)$

In the following paragraphs, we establish the pointwise consistency and weak convergence of the estimator $\hat{\beta}_n(\cdot)$ derived in Section 3.2.2, and present a consistent estimator for its asymptotic variance.

Let $d\tilde{M}_i(u;\beta_0(u)) = d\tilde{N}_i^*(u) - \tilde{Y}_i(u)\exp\{\beta_0(u)' Z_i\}d\Lambda_0(u)\big]$ for $u\in(0,18)$. Thus the expectation $E\big\{d\tilde{M}_i(u;\beta_0(u))\big|Z_i\big\} = 0$ for $u\in(0,18)$. The estimating function $E_n(\gamma;a|\mathbf{B})$ in (9) is

$$\frac{1}{n}\int_0^{18} K_h(u-a)\sum_{i=1}^{n}\left\{Z_i^*(u,a) - \tilde{\bar{Z}}_n^*(\gamma;u,a)\right\}\big[d\tilde{M}_i(u;\beta_0(u)) + \tilde{Y}_i(u)\exp\{\beta_0(u)' Z_i\}d\Lambda_0(u)\big].$$

Similarly as in Web Appendix A.1, we have $\Big\{(n^{-1/2}\sum_{i=1}^n \tilde{M}_i(u;\beta_0),\ n^{-1/2}\sum_{i=1}^n\int_0^u Z_i^*(t,a)d\tilde{M}_i(t;\beta_0))'\Big\}$ converges weakly to a zero-mean Gaussian process with index $u\in(0,18)$, say $(W_{\tilde{M}}, W_{\tilde{M}_z})'$. By the strong law of large numbers, $\tilde{S}_n^{(q)}(\gamma;u,a)$ converges almost surely to $s^{(q)}(\gamma;u,a) = E\Big[Y(u|B_1)Z_1^*(u,a)^{\otimes q}\exp\big\{\gamma' Z_1^*(u,a)\big\}\Big]$ for $q=0,1,2$, and thus $\tilde{\bar{Z}}_n^*(\gamma;u,a)$ to $\bar{z}^*(\gamma;u,a) = s^{(1)}(\gamma;u,a)/s^{(0)}(\gamma;u,a)$ and $\tilde{S}_n^{(2)}(\gamma;u,a)/\tilde{S}_n^{(0)}(\gamma;u,a)$ to $s^{(2)}(\gamma;u,a)/s^{(0)}(\gamma;u,a)$. We can show that $\tilde{E}_n(\gamma_0;a)$ converges a.s. to 0 and so does $E_n(\gamma_0;a)$.

Note that

$$\frac{\partial E_n(\gamma;a)}{\partial\gamma} = -\frac{1}{n}\int_0^{18} K_h(u-a)\left[\frac{\tilde{S}_n^{(2)}(\gamma;u,a)}{\tilde{S}_n^{(0)}(\gamma;u,a)} - \left\{\frac{\tilde{S}_n^{(1)}(\gamma;u,a)}{\tilde{S}_n^{(0)}(\gamma;u,a)}\right\}^{\otimes 2}\right]\sum_{i=1}^{n} d\tilde{N}_i^*(u)$$

and the Taylor expansion of $E_n(\gamma;a)$ about point γ_0 for $\gamma\in\mathcal{B}_\epsilon = \{\gamma : ||\gamma-\gamma_0||\le\epsilon\}$ with $\epsilon>0$:

$$E_n(\gamma;a) = E_n(\gamma_0;a) + \frac{\partial E_n(\gamma_0;a)}{\partial\gamma}(\gamma-\gamma_0) + o\big((\gamma-\gamma_0)\big).$$

We can argue that $E_n(\gamma;a) = 0$ has solutions in $\mathcal{B}_\epsilon$ with the same first component vector, which is $\hat{\beta}_n(a)$. This argument proves the consistency.

Analogous to Section A.1.2 of the Web Appendix, we can show that $\sqrt{nh}(\hat{\gamma}_n-\gamma_0)$ is asymptotically equivalent to $\big\{-\partial E_n(\gamma_0;a)/\partial\gamma\big\}^{-}\big\{\sqrt{nh}E_n(\gamma_0;a)\big\}$. Thus the equivalence yields the asymptotical normality of $\sqrt{nh}(\hat{\beta}_n(a)-\beta_0(a))$ with mean zero and variance matrix $\tilde{AV}(\beta_0;a)$.

The upper left block of $-\partial E_n(\gamma_0;a)/\partial\gamma$ is

$$\hat{\tilde{\Pi}}(\gamma;a) = \frac{1}{n}\int_0^{18} K_h(u-a)\left[\frac{\tilde{S}_{1n}^{(2)}(\gamma;u,a)}{\tilde{S}_{1n}^{(0)}(\gamma;u,a)} - \left\{\frac{\tilde{S}_{1n}^{(1)}(\gamma;u,a)}{\tilde{S}_{1n}^{(0)}(\gamma;u,a)}\right\}^{\otimes 2}\right]\sum_{i=1}^{n} d\tilde{N}_i^*(u)$$

with $\tilde{S}_{1n}^{(q)}(\gamma; u, a) = \sum_{i=1}^{n} \tilde{Y}_i(u) Z_i^{\otimes q} \exp\{\gamma' Z_i^*(u,a)\}/n$ for $q = 0, 1, 2$. Thus $\hat{\Pi}(\hat{\gamma}_n; a)$ is a consistent estimator of $\tilde{\Pi}(\beta_0; a)$.

The first component vector of $\sqrt{nh}\tilde{E}_n(\gamma; a)$ is $n^{-1/2}\sum_{i=1}^{n} \tilde{\tilde{X}}_i(\gamma; a)$ with independent $\tilde{\tilde{X}}_i(\gamma; a) = \int_0^{18} \sqrt{h} K_h(u - a)\{Z_i - s_1^{(1)}(\gamma; u, a)/s^{(0)}(\gamma; u, a)\} d\tilde{N}_i^*(u)$. So $\tilde{\Sigma}(\beta_0; a)$, the limit of the variance of $\sqrt{nh}E_n(\gamma; a)$'s first component vector, is consistently estimated by $\hat{\tilde{\Sigma}}(\hat{\gamma}_n; a) = \sum_{i=1}^{n} \left(X_i(\hat{\gamma}_n; a) - \bar{X}(\hat{\gamma}_n; a)\right)^{\otimes 2}/(n-1)$ with $X_i(\gamma; a) = \int_0^{18} \sqrt{h} K_h(u - a)\big[Z_i - \tilde{S}_{1n}^{(1)}(\gamma; u, a)/\tilde{S}_n^{(0)}(\gamma; u, a)\big] d\tilde{N}_i^*(u)$ and $\bar{X}(\gamma; a) = \sum_{i=1}^{n} X_i(\gamma; a)/n$.

Thus we obtain a consistent estimator for $\tilde{AV}(\beta_0; a)$: $\hat{\tilde{\Pi}}(\hat{\gamma}_n; a)^{-1}\hat{\tilde{\Sigma}}(\hat{\gamma}_n; a)\hat{\tilde{\Pi}}(\hat{\gamma}_n; a)^{-1}$.

A.3 Asymptotics of $\tilde{\beta}_{nW}(\cdot)$ and $\bar{\tilde{\beta}}_{nW}(\cdot)$ in Section 3.2.2

A.3.1 Estimator $\tilde{\beta}_{nW}(\cdot)$ by Procedure A

Note that, with a fixed n, $\bar{U}_{n,W}(\gamma; a) \to E_n(\gamma; u)$ a.s. and $Var\{\bar{U}_{n,W}(\gamma; a)\} \to Var\{E_n(\gamma; u)\}$ as $W \to \infty$. Thus $\partial \bar{U}_{n,W}(\gamma; a)/\partial\gamma$ and $\partial E_n(\gamma; u)/\partial\gamma$ have the same limit a.s. as $n, W \to \infty$. Those limits imply that $\tilde{\beta}_{nW}(\cdot)$ is asymptotically equivalent to $\hat{\beta}_n(\cdot)$ as $n, W \to \infty$.

Further, it results in a consistent estimator for the variance of $\tilde{\beta}_{nW}(a)$ to approximate $\tilde{Y}_i(\cdot)$ and $d\tilde{N}_i^*(\cdot)$ used in the consistent estimator for $\tilde{AV}(\beta_0; a)$ in Web Appendix A.2 by $\hat{\tilde{Y}}_i(u) = \frac{1}{W}\sum_{w=1}^{W} Y(u|B_i^{(w)})$ and $d\hat{\tilde{N}}_i^*(u) = \frac{1}{W}\sum_{w=1}^{W} Y(u|B_i^{(w)}) dN_i(u|B_i^{(w)})$, respectively.

A.3.2 Estimator $\bar{\tilde{\beta}}_{nW}(\cdot)$ by Procedure B

Note that, as $n \to \infty$ with fixed W, $\sqrt{nh}(\bar{\tilde{\gamma}}_{nW} - \gamma_0)$ is asymptotically equivalent to $\sum_{w=1}^{W}\left\{\left(\partial U_n(\gamma_0; a\big|\mathbf{B}^{(w)})/\partial\gamma\right)^{-}\sqrt{nh}U_n(\gamma_0; a\big|\mathbf{B}^{(w)})\right\}/W$. That together with the asymptotic properties of $\partial U_n(\gamma_0; a\big|\mathbf{B})/\partial\gamma$ and $\sqrt{nh}U_n(\gamma_0; a\big|\mathbf{B})$ yields the asymptotic equivalence of $\bar{\tilde{\beta}}_{nW}(\cdot)$ with $\hat{\beta}_n(\cdot)$ as $n, W \to \infty$.

The variance of $\bar{\tilde{\beta}}_{nW}(\cdot)$ can be consistently estimated by

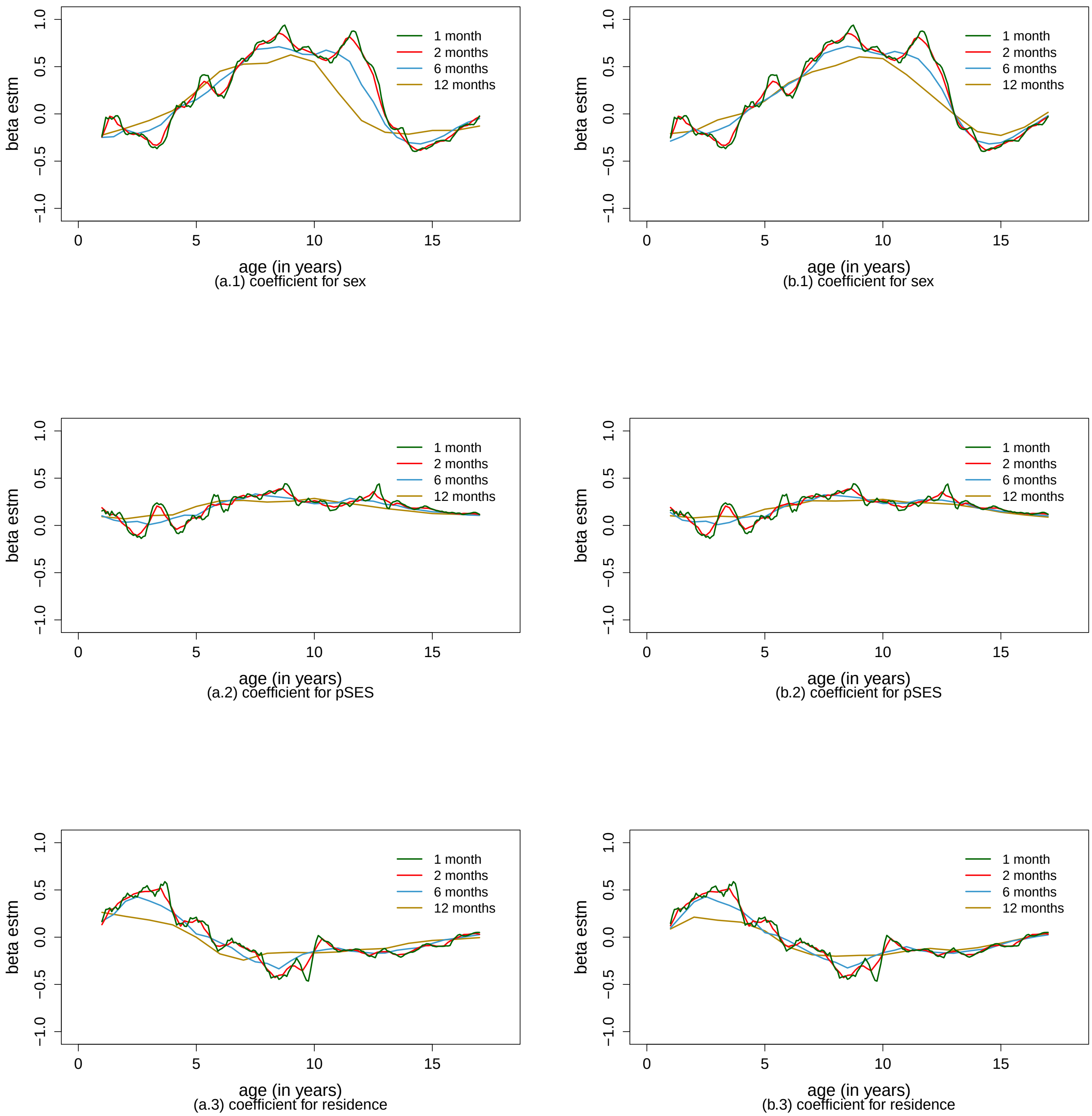
$$\frac{1}{W}\sum_{w=1}^{W}\widehat{AV}(a|\mathbf{B}^{(w)}) + \frac{1}{W}\sum_{w=1}^{W}\left\{\hat{\beta}_n(a|\mathbf{B}^{(w)}) - \bar{\tilde{\beta}}_{nW}(a)\right\}\left\{\hat{\beta}_n(a|\mathbf{B}^{(w)}) - \bar{\tilde{\beta}}_{nW}(a)\right\}',$$

where $\widehat{AV}(a|\mathbf{B})$ is the variance estimator for $\hat{\beta}(a|\mathbf{B})$ in Web Appendix A.1.

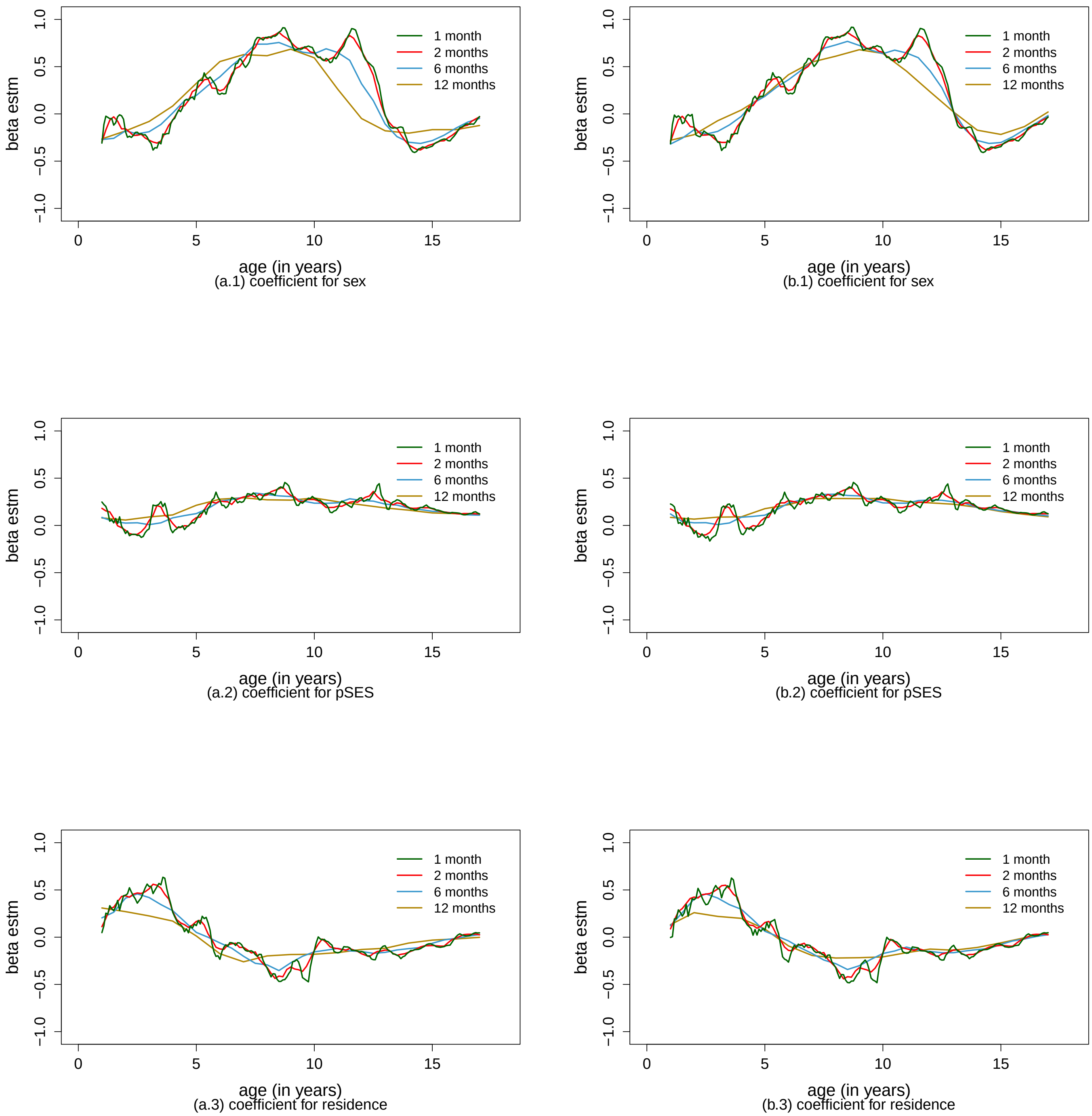
Web Appendix B: Additional Numerical Studies

B.1 Analyses of PMHC Data with Different Time Units

Web Figure 1 and Web Figure 2 present the curve estimates by Procedures A and B under different time units (1, 2, 6, and 12 months), respectively.



Web Figure 1: **Regression Parameter Estimates with PMHC Data under the Marginal Model by Procedure A of Section 3.2.2 with Different Time Units: the left/right panels are the local constant/linear estimates**



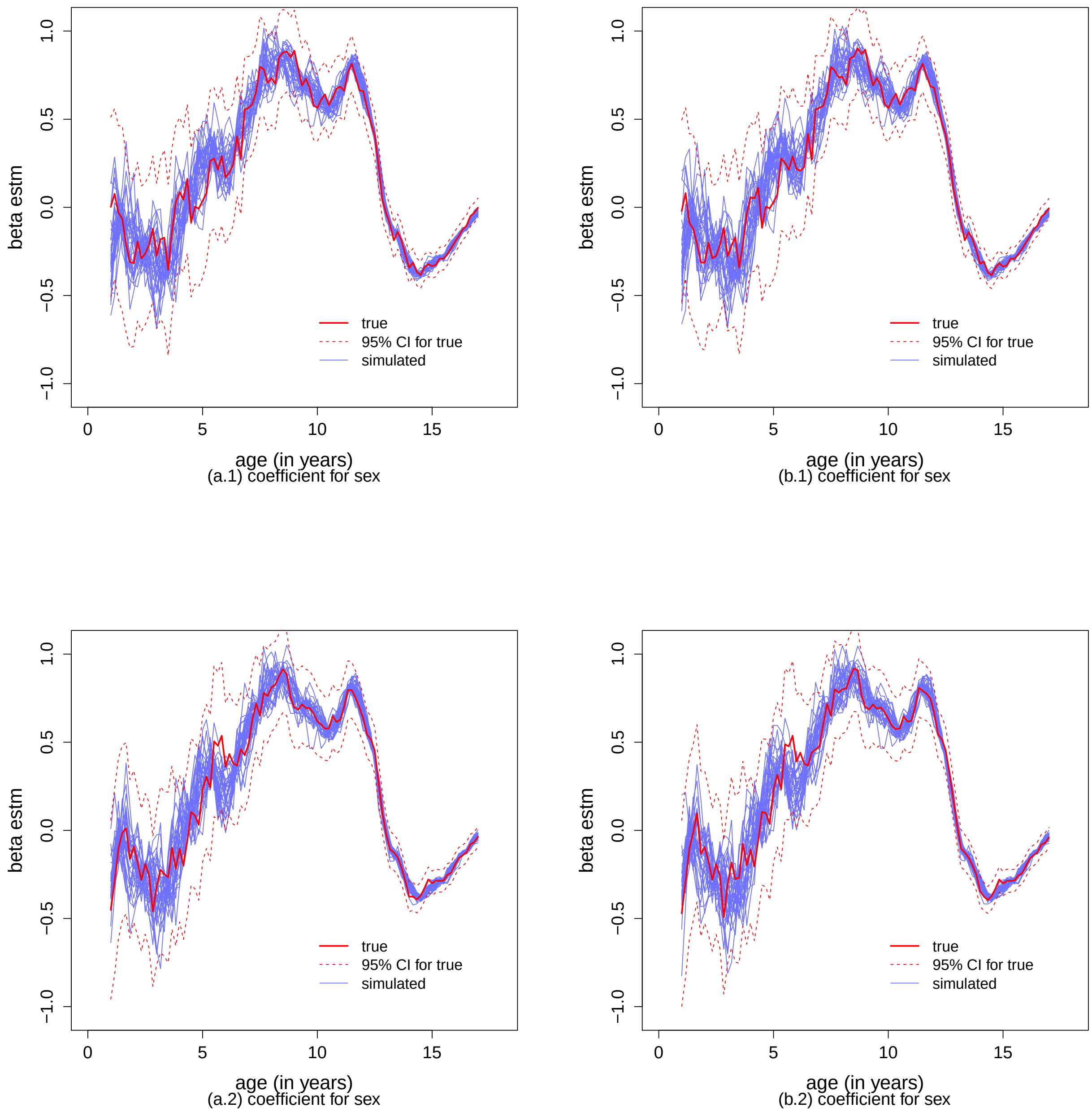
Web Figure 2: **Regression Parameter Estimates with PMHC Data under the Marginal Model by Procedure B of Section 3.2.2 with Different Time Units: the left/right panels are the local constant/linear estimates**

B.2 Simulation

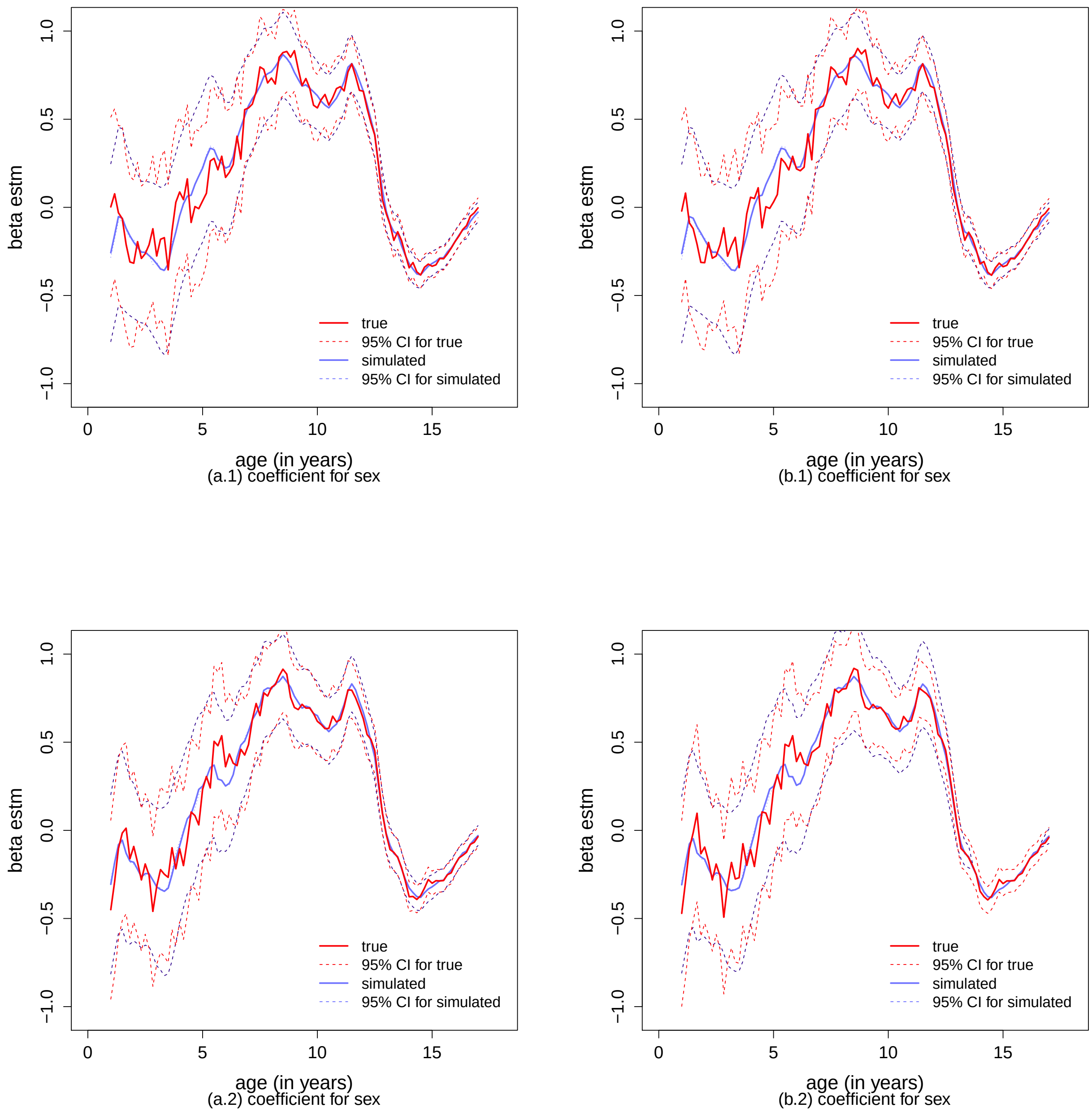
We conducted a simulation study to examine the proposed procedures in Section 3.2.2. The simulation used the PMHC dataset with one covariate (sex) and generated one birthdate set as the "true" birthdates. Using the equivalent settings of Section 4 (time unit of 2 months, h=3), the realization of the estimator in Section 3.2.1 was considered to be the "true" curve.

To show the variability in the estimates that arise from the generation of birthdates, we obtained 30 estimated curves from Procedures A and B from Section 3.2.2 with $W = 1$. These estimated curves appear in Web Figure 3 along with the "true" estimate and its associated approximate 95% confidence interval (CI). These estimated curves show the same general features of the "true" curve and the 95% CIs contain nearly all curves.

To further investigate the procedures, we also obtained 500 estimated curves using the local constant and local linear estimators of Section 3.2.2 by Procedures A and B with $W = 100$. We call these 500 estimated curves simulations. Web Figure 4 shows the estimate of the "true" curve and mean over the simulations. In addition, the associated approximate 95% confidence intervals are provided for the "true" curve and the simulation mean. For example for Procedure A, the variance for the mean of the 500 simulations $\{\tilde{\beta}_{nW,1}(\cdot), \ldots, \tilde{\beta}_{nW,500}(\cdot)\}$ is calculated by $\sum_{j=1}^{500} \widehat{Var}(\hat{\tilde{\beta}}_{nW,j}(\cdot))/500 + Var(\{\hat{\tilde{\beta}}_{nW,j}(\cdot)\}_{j=1}^{500})$. Both the local constant and local linear simulations capture the general pattern of the "true" curve and the 95% approximate confidence intervals contain the "true" curve. The mean of the simulations is smoother, as expected, because it is based on the mean of estimated curves.



Web Figure 3: **Selected Regression Parameter Estimates from Simulation Studies and True Generated Data with Approximate 95% Confidence Intervals Using Procedure A (top row) and Procedure B (bottom row): the left/right panels are the local constant/linear estimates**



Web Figure 4: **Regression Parameter Estimates from Simulation Studies and True Generated Data with Approximate 95% Confidence Intervals using Procedure A (top row) and Procedure B (bottom row): the left/right panels are the local constant/linear estimates**

References

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