Textbook Reading: Chapters 3, 4 **Due Date:** Friday, January 29, 2021 by 11:59pm

Instructions

- Upload a copy of your assignment (pdf format) to the Crowdmark link you've received via email.
- *Correctness, Clarity, & Conciseness* of presentation are reflected in the grading.
- Collaborative discussion on the assignment in encouraged, but the write-up should reflect you own understanding & results. Acknowledge colleagues, TA, or other assistance you received.

Questions

1. Find the inverse of each of the following permutations. Verify it is the inverse by computing the product and showing it is the identity permutation.

   (a) $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ (b) $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 7 & 5 & 2 & 3 & 8 & 6 \end{pmatrix}$

2. Consider the following permutations in array form

   $$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 2 & 3 & 7 & 1 & 8 & 5 & 4 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 6 & 7 & 1 & 3 & 8 & 2 \end{pmatrix}, \quad \gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 7 & 6 & 5 & 4 & 3 & 2 & 8 \end{pmatrix}.$$

   Determine each of the following.

   (a) $\alpha\beta$ (b) $\alpha\gamma\beta$ (c) $\alpha^{-1}\gamma\alpha$

3. Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 5 & 3 \end{pmatrix}$.

   (a) What is α^{42}?
   (b) Explain how you know $\alpha^{2021} \neq \varepsilon$, without actually computing all 2021 powers of α.

4. Show that for any $\alpha \in S_n$, $\text{ord}(\alpha) = \text{ord}(\alpha^{-1})$.

5. (a) For any permutations α and β and any integer n show that $(\alpha^{-1}\beta\alpha)^n = \alpha^{-1}\beta^n\alpha$.
   (b) Use the result of part (a) to conclude that β and $\alpha^{-1}\beta\alpha$ have the same order.

6. Show that if $\alpha\beta\gamma\beta^{-1}\alpha = \alpha\beta\sigma\beta^{-1}\alpha$ then $\gamma = \sigma$.
   Hint: Use the cancellation property.

7. Consider the following permutations in S_{10}

   $$\alpha = (3\ 7\ 4), \qquad \beta = (5\ 10\ 6)(2\ 9\ 4\ 7)(3\ 8)$$

   Determine each of the following:

   (a) $\alpha\beta$ (b) α^{-1} (c) $\text{ord}(\alpha)$ (d) $\text{ord}(\beta)$

8. **There is always something that doesn't commute.** Show that if $n \geq 3$, then for every element α in S_n, if α is not the identity permutation ε, then there is some other permutation β in S_n with which α does not commute: $\alpha\beta \neq \beta\alpha$.

9. What is the order of the product of three disjoint cycles of lengths 6, 12 and 26?

10. Show S_5 contains no element of order 7.

11. Let $\alpha = (1\ 7\ 4\ 5\ 9)(3\ 8)(10\ 6\ 2)$. If α^m is a 3-cycle, what can you say about m?

12. (a) Give an example of permutations α and β such that ord$(\alpha) = 3$, ord$(\beta) = 3$, and ord$(\alpha\beta) = 5$.
(b) Give an example of permutations α and β such that ord$(\alpha) = 3$, ord$(\beta) = 3$, and ord$(\alpha\beta) = 10$.

13. Show that the number of elements α in S_n such that $\alpha^3 = \varepsilon$ is odd. In other words, show the set

$$\{\alpha \in S_n \mid \alpha^3 = \varepsilon\}$$

has odd cardinality.