Textbook Reading: Chapters 10, 11 **Due Date:** Friday, March 5, 2021 by 11:59pm

Instructions

- Upload a copy of your assignment (pdf format) to the Crowdmark link you've received via email.
- *Correctness, Clarity, & Conciseness* of presentation are reflected in the grading.
- Collaborative discussion on the assignment in encouraged, but the write-up should reflect you own understanding & results. Acknowledge colleagues, TA, or other assistance you received.

Questions

1. (a) List the element of $\mathbb{Z}_6$, and write out the Cayley table for this group.
   (b) List the element of $U(10)$, and write out the Cayley table for this group.
2. (a) With pictures and words, describe each symmetry in the dihedral group D_5 (the set of symmetries of a regular pentagon).
   (b) Write out a complete multiplication (Cayley) table for D_5.
   (c) Is D_5 abelian (that is, does every element commute with every other element)?
   (d) Determine all the subgroups of D_5.
3. For any $n \geq 3$, is D_n a cyclic group? That is, does $D_n = \langle g \rangle$ for some $g \in D_n$?
4. Determine which elements of $\mathbb{Z}_{22}$ are generators for $\mathbb{Z}_{22}$. That is, find all $g \in \mathbb{Z}_{22}$ such that $\mathbb{Z}_{22} = \langle g \rangle$.
5. List all the elements of order 6 in $\mathbb{Z}_{600}$.
6. Find all the subgroups, and determine generators for each subgroup, for each of the following.
   (a) $\mathbb{Z}_{12}$
   (b) $\mathbb{Z}_{17}$
7. (a) Find a subgroup of order 4 in S_4.
   (b) Find a subgroup of order 8 in S_4. (Hint: don't try a brute force approach here where you try to build the group up... instead think if you know of a group of order 8 that lives inside S_8.)
8. Suppose that G is a cyclic group and 10 divides $|G|$. How many elements of order 10 does G have? If a is one element of order 10, list the other elements of order 10.
9. Show that if G is a group where $|G| = p$ is prime then G is cyclic.
10. Prove that if G is a group with the property that the square of every element is the identity (i.e. every element has order 2), then G is abelian.
11. Let $|G| = 33$. What are the possible orders for the elements of G? Without using Cauchy's Theorem, show that G must have an element of order 3.