



SFU

COURSE OUTLINE  
Spring 2021



**MATH304: Mathematical Journeys IV**  
**Permutation Puzzles: A Mathematical Perspective**

## General information

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-  *Course:* MATH304: Mathematical Journeys IV  
Permutation Puzzles: A Mathematical Perspective
-  *Credits:* 3 credits
-  *Instructor:* Dr. Jamie Mulholland
-  *Course website:* [www.sfu.ca/~jtmulhol/permutationpuzzles](http://www.sfu.ca/~jtmulhol/permutationpuzzles)
-  *Lectures:* MWF 10:30-11:20
-  *Prerequisite:* Integral Calculus (MATH152 or 155 or 158) and Linear Algebra (MATH 232 or 240). Previous computing experience is not required, but can be helpful if you have it.
-  *email:* [j\\_mulholland@sfu.ca](mailto:j_mulholland@sfu.ca)
-  *office:* online
-  *Teaching Assistant:* TBA

## Learning Outcomes

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### Overall outcomes:

To appreciate mathematics as a rich, diverse, and creative subject whose applications are limited only by one's imagination. When, in casual conversation, presented with the question: "I never understood mathematics, what is it good for?", you will be able to come to the defence of mathematics with various fun, insightful, and interesting examples in support of the claim that mathematical concepts have applicability far beyond the context in which they were developed.

By the end of this course students will be able to:

### Puzzles:

-  model and investigate puzzles such as a Rubik's cube, and the 15-puzzle by applying the algebraic theory of permutations;
-  analyze the solvability of the various puzzles under consideration in this course;
-  create their own set of move sequences for solving Rubik's cube;
-  understand the definition of the term *permutation puzzle*;
-  identify instances of permutation puzzles and model them in a computer algebra system;
-  apply concepts such as Gaussian elimination, rank, and null space from Linear Algebra to analyze the solvability of the Lights Out puzzle;

### Permutations:

-  understand the algebraic definition of the term permutation and be able to convert between the following representations of a permutation: cycle form, array form, arrow diagram, cycle-arrow diagram;

- 🧩 perform algebraic operations in the symmetric group: multiplication, conjugation, commutators; and find the inverses, order, and parity of a permutation;
- 🧩 decompose a permutation into a product of 2-cycles; decompose an even permutation into a product of 3-cycles; decompose a permutation into a product of 3-cycles and a single 2-cycle;
- 🧩 prove statements concerning the theory of permutations;

### Group Theory:

- 🧩 identify some basic types of finite groups and their subgroups;
- 🧩 perform algebraic operations in the following finite groups: symmetric group, cyclic groups, unit group modulo  $n$ , dihedral groups;
- 🧩 apply group theory to count like a *professional*. Specifically,
  - 🧩 determine the number of symmetries of various geometric objects (for example, the number of rotational symmetries of a soccer ball) - *Orbit Stabilizer Theorem*.
  - 🧩 determine the number of different ways there are to design various geometric objects (for example, the number of ways there are to paint the sides of a cube so that three faces are white and three are black) - *Burnside's Theorem*
- 🧩 prove statements concerning the structure of groups and their subgroups.

### SageMath - Computer Algebra System:

- 🧩 perform algebraic operations in the symmetric group using the computer algebra system SageMath;
- 🧩 explore the theory of permutations and create conjectures through experimentation
- 🧩 use the tools of linear algebra over a finite field: solve a linear system, compute the rank, null space, eigenvalues, eigenvectors of a matrix
- 🧩 perform basic programming in Python/SageMath: sets, lists, conditionals, while loops, for loops, list comprehensions, user defined functions

### Problem Solving:

- 🧩 identify the common theme that links all problem solving techniques encountered in mathematics
- 🧩 honestly say they had lots of fun solving a variety of problems in a university mathematics class!

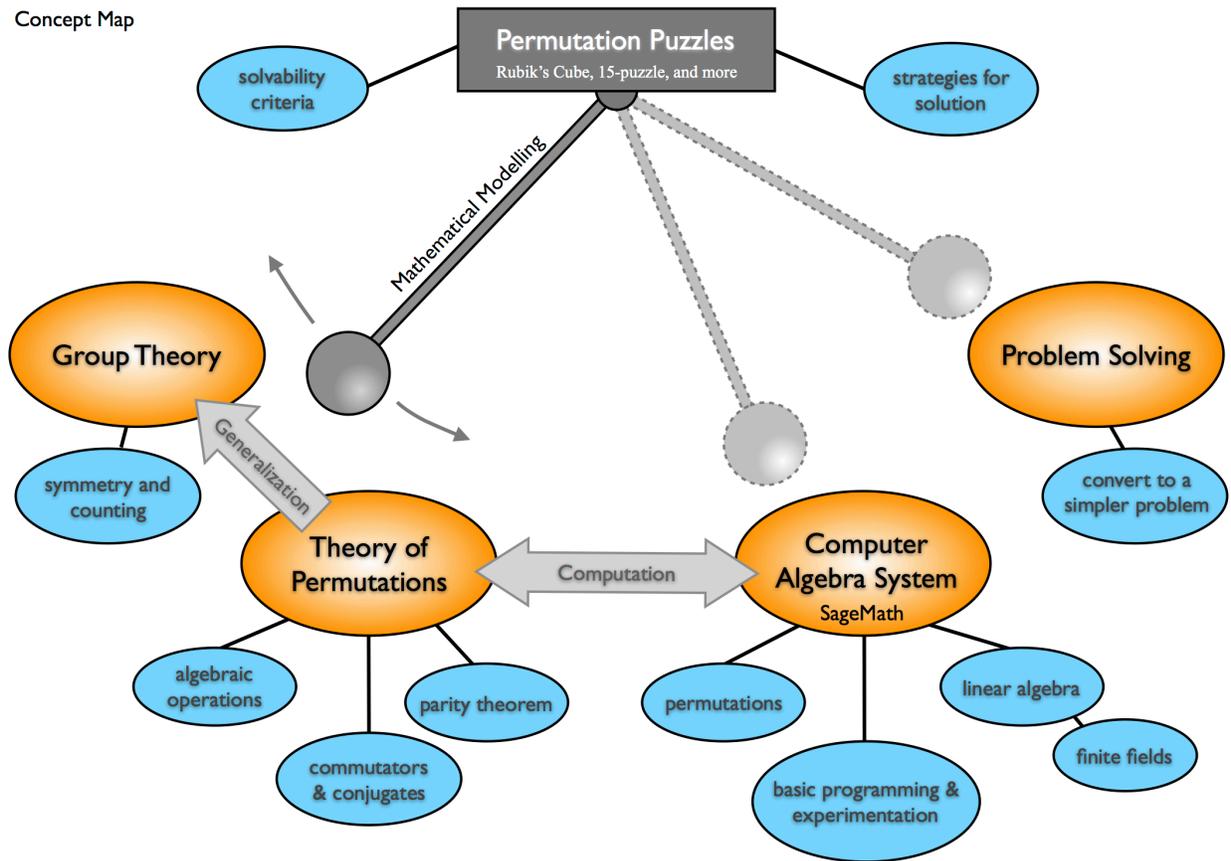
## Instructional Method

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There will be 3 hours of lectures per week, over 13 weeks. In class activities will includes: lectures, group work, and hands-on activities.

# Course Content

Concept Map



## Course Materials

- 📖 Textbook: *Permutation Puzzles: A Mathematical Perspective*. Jamie Mulholland. Available as a downloadable pdf from the course website.
- 📖 Your own Rubik's Cube. See course website for suggestions on where to purchase.
- 📖 Software: SageMath. Free cloud based accounts at <https://cloud.sagemath.com>, or free download from <http://www.sagemath.org/>.
- 📖 Printable Swap and 15-puzzle (downloadable pdf from course website).

## Assignments and Evaluation

There will be 10 weekly homework assignments consisting of roughly 12 question per assignment that are mostly taken from the exercise in the course text book.

Assignments	30%
Midterms	50% (two - each worth 25%)
Final Project	20%

## Further Information

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If you have a disability, please contact the instructor to arrange a time to discuss your situation. It would be helpful if you contact the SFU Centre for Students with Disabilities at 778.782.3112 <http://students.sfu.ca/disabilityaccess.html> before you do this.

It is important to familiarize yourself with the policies and guidelines pertaining to academic integrity (cheating, plagiarism, etc.) Please refer to SFU's Code of Academic Integrity and Good Conduct for Student Discipline (S 10.02) (<http://www.sfu.ca/policies/gazette/student/s10-02.html>)

Additional policies governing academic issues that affect students can be found in SFU's Human Rights Policy (GP 18) (<http://www.sfu.ca/policies/gazette/general/gp18.html>)