Math 5320

Homework 9

From the book, do Ch. 15: 2.3, 3.2, 3.3, 3.4(a-c), 4.2(a-d). In addition, do the following problems, which are meant to help with solving some of the book problems.

1. (This may help with 2.3) Let $K, K'/\mathbb{Q}$ be field extensions. Recall that if $p(x) \in \mathbb{Q}[x]$ has a solution $\alpha \in K/\mathbb{Q}$ and there is an isomorphism $K \to K'$ that sends elements of Q to themselves, then the image of α under this isomorphism is also a root of $p(x)$. Show that similarly, if a polynomial in multiple variables $p(x_1, \ldots, x_n) \in \mathbb{Q}[x_1, \ldots, x_n]$ has a solution in K, then it has a solution in K' as well.

   Another remark that may help with 2.3: It is impossible for $x_1^2 + \cdots + x_k^2 = -1$ to have any real solutions.

2. (This may help with 3.2) Let α be a root of the polynomial $x^4 + 3x + 3$ in $\mathbb{C}$. What is $[\mathbb{Q}(\alpha, \sqrt[3]{2}) : \mathbb{Q}]$?