Worksheet 22: Degrees of field extensions, adjoining more than one element to a field.

1. Let $F \subseteq K$ be a field extension. Why is it true that $K \cong F$ iff $[K: F]=1$ ?
2. Let $F \subseteq K$ be a field extension and $\alpha, \beta \in K$ algebraic over $F$. Recall how we constructed $F(\alpha)$. How would we then go about constructing $(F(\alpha))(\beta)$ ?
3. What is $[\mathbb{Q}(\sqrt{2}, i), \mathbb{Q}]$ ? What is the degree of $i$ over $\mathbb{Q}(\sqrt{2})$ ?
