Math 5320, 3/14/18 Name:

Worksheet 24:

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1. For any ring $R$, there is an action of the symmetric group $S_n$ on $R[x_1, \ldots, x_n]$ permuting the variables. A polynomial in $R[x_1, \ldots, x_n]$ that is fixed by every permutation in $S_n$ is called *symmetric*.

   Why does the set of symmetric functions in $R[x_1, \ldots, x_n]$ form a subring?

2. The *elementary symmetric functions* in $R[x_1, \ldots, x_n]$ are defined to be

$$\begin{aligned} s_1 &:= x_1 + x_2 + \cdots + x_n \\ s_2 &:= \sum_{i<j} x_i x_j \\ &\vdots \\ s_n &:= x_1 \cdots x_n \end{aligned}$$

   The Symmetric Functions Theorem (Theorem 16.1.6) tells us that any symmetric polynomial in $R[x_1, \ldots, x_n]$ can be written as a polynomial in the elementary symmetric functions, that is, as $p(s_1, \ldots, s_n) \in R[s_1, \ldots s_n] \subset R[x_1, \ldots, x_n]$.

   Write the symmetric polynomial $x_1^2 + x_2^2 + x_3^2 \in \mathbb{Q}[x_1, x_2, x_3]$ as a polynomial in the elementary symmetric functions $s_1, s_2, s_3$.

3. Using the proof of the Primitive Element theorem, what are some possible primitive elements for the extension $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ of $\mathbb{Q}$?

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