Worksheet 33: Unsolvability of the quintic

1. Last time we proved that $G(\mathbb{Q}(\sqrt{2}, \sqrt{3}) / \mathbb{Q}) \cong(\mathbb{Z} /(2) \times \mathbb{Z} /(2),+)$ where and the subroup corresponding to $\mathbb{Q}(\sqrt{6})$ is the group containing $(0,0)$ and $(1,1)$.
Is $\mathbb{Q}(\sqrt{6}) / \mathbb{Q}$ Galois? What are some different methods you could use to answer this question?
2. Suppose we have a polynomial $p(x) \in F[x]$ and let $K$ be its splitting field. Show that the degree of any irreducible factor of $p(x)$ divides $[K: F]$ and that no prime factor of $[K: F]$ can be bigger than the degree of any irreducible factor of $p(x)$.
