Worksheet 7: Chinese Remainder Theorem, Maximal Ideals

1. In the problem set that was due today, you proved that if $R$ is a ring with ideals $I$ and $J$ such that $I+J=R$ and $I J=0$, then the following map is a ring isomorphism:

$$
R \rightarrow R / I \times R / J, \quad r \mapsto(r+I, r+J)
$$

Use this result to prove that $\mathbb{Z} /(12)$ is isomorphic to $\mathbb{Z} /(3) \times \mathbb{Z} /(4)$.
2. What are the maximal ideals of $\mathbb{Z}$ ?
3. Any questions about the upcoming midterm?

