Worksheet 9: Integral domains, Euclidean domains

1. Show that cancellation works in integral domains: if $R$ is an integral domain and $a, b, c \in R$ such that $a c=b c$, show that $b=c$.
2. Show that if $R$ is an integral domain, the degree of the product of two polynomials $f(x), g(x)$ in $R[x]$ is the sum of the degrees of $f$ and $g$. Find an example of a ring $R$ that isn't an integral domain and some polynomials with coefficients in $R$ that don't obey this rule!
3. Show that if $R$ is an integral domain then so is $R[x]$.
4. Show that when you divide $f(x)=x^{2}+\frac{1}{2}$ by $x-3$ in $\mathbb{Q}[x]$, the remainder is the constant polynomial $f(3)$.
