Build a 4-Dimensional Hypercube!

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Splash

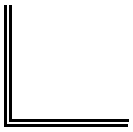
The hypercube we'll build is a 3-D model of the fourth dimension,
just like a flat drawing of a cube is a 2-D model of the third dimension.
Read page 3 for a quick explanation of hyperspace, or jump right in and start building!

1) Make sure you have:
- **6 long** (12 inch) **orange** pipe cleaners
- **6 long** (12 inch) **black** pipe cleaners
- **8 short** (6 inch) **green** pipe cleaners

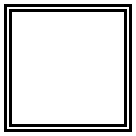
(If you have 4 long (12 inch) green pipe cleaners, bend them in half and cut at the half-way point.)

You can use a different color combination if you want.

2) Take the **long orange** pipe cleaners out of the bag. Bend each one in half, then bend it back so that is forms a right (90°) angle, making an **L shape** like this:

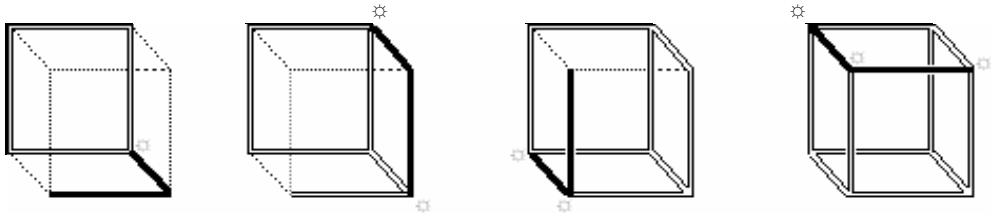


3) Attach two of the L's together to make a **flat square**:



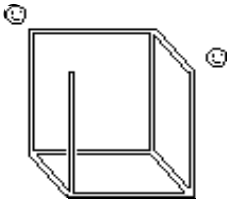
TIP FOR ATTACHING PIPE CLEANER ENDS:
Bend a very short length of the ends around each other and press down firmly.

4) Attach the other L's together by according to the pictures below.
The ☼ show where an attachments are made.
Attaching in a different way may not result with a cube, so follow the pictures carefully.



Now you should have a cube!
Check the corners to make sure everything is attached well.
Put this cube aside for now.

5) Using the **long black** pipe cleaners, repeat steps 2, 3, and 4, *except for the last L* attachment. You should then have:



Notice that there is one leg standing up attached at only one end.
In the picture above, the corners not fully attached are labeled with ☺.

6) Lower the completed **orange cube** over the **black leg** that's only attached at one end, so that this **black leg** goes through the *middle* of the **orange cube**.

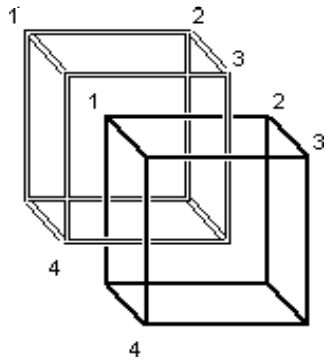
7) Attach the **last black L**'s *corner* to the *top* of the unattached **black leg**.

8) Push the ends of the **black L** through the middles of the **orange cube's square "faces."**

9) Attach the ends of the **black L** to the ☺ black corners to complete the black cube.

10) **Now you should have two cubes within each other.**
They shouldn't be able to come apart. Check that all your attachments are holding.

11) Take the **short green** pipe cleaners, and connect each corner of the **black cube** to the **same corner** of the **orange cube**. For example, following the picture below, attach black corner 1 to orange corner 1, black corner 2 to orange corner 2, and so on.



10) **You're done!** Remember to check that the attachments are all secure.
Pipe cleaners are flexible so play around (don't bend too much, though).
A hypercube looks great hanging on a string from your ceiling at home,
or on your head as a zany hat!

Thinking About Dimensions

What are **dimensions**? One way to think of it is how many (perpendicular) directions we can move in.

For example, we live in a **3-D** (3-Dimensional) **world**, and can move in 3 ways:
1) up-down, 2) left-right, 3) back-forth.

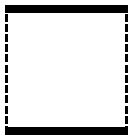
This piece of paper is 2-D, because it only has up-down and left-right as directions.
(It actually is 3-D, because the paper has thickness in the back-forth direction,
but when the paper is flat, it's so small that we can ignore it.)

So how do you get from one dimension to a higher one?

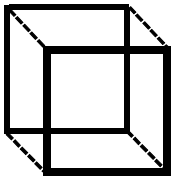
Let's start with the **0-dimension**, a **point**: **.**

Now connect that point to another point, forming a **line**. This is **1-dimensional**.

Connect the line to another line, making a **2-D square**:



Connect the square to another square to form a **3-D cube**:



Notice that the figure to the right is not *actually* a 3-D cube, but a *2-D picture* of a cube.
This paper is 2-D and can't actually show higher-dimension objects (like a 3-D cube) exactly.

Now what happens if you connect a cube with another cube?
You get a 4-D hypercube!
A 2-D picture of a 4-D hypercube is messy (try drawing it!),
but we can build a pretty good model in 3-D. Let's try!

Some interesting questions to think about as you're building your hypercube:

* Why doesn't moving diagonally count as another dimension in our world?
* Why do you think "time" is sometimes called the fourth dimension in our world?
* Why can't a 4-D hypercube be built in our world with solid sides, not just a wire frame?
* Does a 4-D hypercube have volume?
  What does "volume" mean? How is it different from or similar to "area" and "length"?
* What would we get if we connected two 4-D hypercubes together?
* How would a person living in a 4-dimensional world see your 4-D hypercube?
  What about a person living in a 5-dimensional world?

Interested in Hyperspace?

Here are some cool resources to check out.

What is the Fourth Dimension? by Eric Saltsman
A great discussion of the fourth dimension, without too much math.
http://www.geocities.com/CapeCanaveral/7997/whatis4d.html

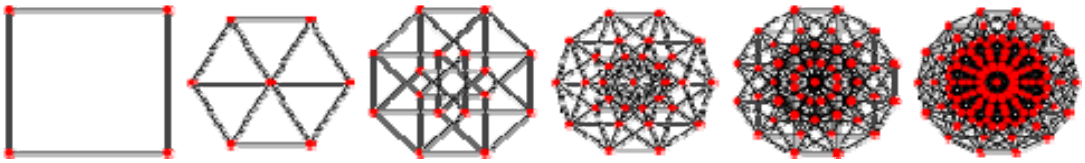
Flatland: A Romance of Many Dimensions, by Edwin Abbot
A classic story about life in a 2-D world. Lots of fun!
http://www.alcyone.com/max/lit/flatland/

The Fourth Dimension: A Guided Tour of the Higher Universes, by Rudy Rucker
A fun, illustrated exploration of what a 4-D world would be like.

Animated Hypercube Rotations, by Andrew Hamilton
A neat animation of a rotating hypercube
http://casa.colorado.edu/~ajsh/sr/hypercube.html

Hypercube Rotations, by Drew Olbrich
Good pictures of hypercube rotations in space
http://www.traipse.com/hypercube/

Hypercube, from Eric Weisstein's World of Mathematics
A very technical description of some of the math behind hypercubes.
http://mathworld.wolfram.com/Hypercube.html



2-D Square | 3-D Cube | **4-D Hypercube** | 5-D | 6-D | 7-D

Image courtesy of MathWorld.Wolfram.com