1 Pascal's triangle

1. Multiply to find the following polynomials:
   (a) $(x+1)^3$
   (b) $(x+1)^4$
   (c) $(x+1)^5$
   (d) $(x+2)^3$
   (e) $(x-1)^3$
2. How many ways are there to choose 2 marbles from a bag of 5 marbles?
3. How many ways are there to choose 3 marbles from a bag of 5 marbles?
4. How many ways are there to choose 4 marbles from a bag of 5 marbles?

2 Roots of unity and the complex plane

Some more cool examples of where cyclic groups arise are *roots of unity*, which live inside the complex plane.

1. Find all the solutions to the equation $x^2 = 1$, that is, all the "square roots of unity". Check that these form a group under multiplication.
2. Find all the solutions to the equation $x^3 = 1$ (hint: you will need the complex numbers). Check that these form a group under multiplication.
3. Find all the solutions to the equation $x^4 = 1$. Check that these form a group under multiplication.
4. Graph the solutions to each of the previous problems. What do they look like?
5. Use Euler's formula $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ to write your solutions to the previous questions as complex exponentials. What patterns do you notice?
6. Can you express what all the n-th roots of unity will look like in terms of complex exponentials?
7. Do the cube roots of unity form a group under multiplication? Do the fourth roots of unity form a group under multiplication? Do the n-th roots of unity form a group under multiplication?
8. How many generators will these groups have?