Math 817, Fall 2023, Dr. Honigs
Homework 2, Part 1 (of two parts)
All of Homework 2 is due Tues. **Oct. 3** at the start of class

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Instructions:

- You are encouraged to work in groups. However, write your solutions in your own words.
- In your solutions, list any names of people you have worked with. (This is practice for listing collaborators!)
- In your solutions, refrain from using language like "clear", "obvious", or "easy".

Homework 2 Part 1 will be graded out of 50 points. There are 9 textbook questions and one additional question. All of them are worth 5 points each.

Textbook questions:

Aluffi Ch. II

- 6.4, 6.9 (Cyclic groups)
- 6.10, 7.5 ($\mathrm{SL}_2(\mathbb{Z})$ and the modular group)
- 7.1, 6.16, (S_3)
- 6.7, 7.11, 7.14 (Relating to inner automorphisms, the commutator)

Remarks:

- To solve 6.9 you will likely want to use Bézout's lemma, which we proved in Exercise 2.13. You may use the following (slightly upgraded) version of the statement now and in the future: For any integers m and n with greatest common divisor d, there exist integers a and b so that $am + bn = d$.
- I recommend solving 7.1 before 6.16.

Additional question:

1. Let $\mathcal{C}$ be a category and $A, B \in \mathrm{Obj}(\mathcal{C})$. Suppose the product of A and B exists in $\mathcal{C}$. That is, there is an object $A \times B$ and morphisms $\pi_A : A \times B \to A$, $\pi_B : A \times B \to B$ satisfying the universal property given in section 5.4 of Ch. 1.
   (a) Prove the following statement: For any $D \in \mathrm{Obj}(\mathcal{C})$, there is a bijection
   $$\mathrm{Hom}_{\mathcal{C}}(D, A) \times \mathrm{Hom}_{\mathcal{C}}(D, B) \to \mathrm{Hom}_{\mathcal{C}}(D, A \times B)$$
   (b) Suppose the coproduct of A and B also exists. There is a bijection for coproducts that is analogous to the statement for products in part (a). What is it?