Math 817, Fall 2023, Dr. Honigs
Homework 2, Part 2 (of two parts)
All of Homework 2 is due Tues. **Oct. 3** at the start of class

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**Instructions:**

- You are encouraged to work in groups. However, write your solutions in your own words.
- In your solutions, list any names of people you have worked with. (This is practice for listing collaborators!)
- In your solutions, refrain from using language like "clear", "obvious", or "easy".

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**Textbook questions**:

Aluffi Ch. II

- 8.7 (coproduct, free groups)
- 8.17 (order $p$ subgroup of an abelian group)
- 8.21 (review of 2nd iso. thm)
- 8.22, 8.23, 6.11 (cokernels, more with $S_3$)
- 9.14 (action of modular group)

Aluffi Ch. IV

- 1.5 (inner automorphisms)

**Remarks**:

- In several problems we need the fact that it's possible to produce a "smallest" *normal* subgroup containing a set (reference: Ch. 2, §8.2, also §6.3). Given a set $S$ of elements in a group $G$, we obtain the smallest normal subgroup containing $S$ by taking the intersection of all the normal subgroups of $G$ that contain $S$.
- 8.7: $R$ is defined to be the smallest normal group containing $\mathscr{R}$, which is not exactly the same as being generated by $\mathscr{R}$. In writing your solution, you may find it helpful to prove the following fact: Given a group homomorphism $F(A) \to G$, if all of the elements of $\mathscr{R}$ map to $e_G$, then so do all the elements of $R$.
- 8.17: The assumption that $G$ is abelian is necessary to make the proposed argument work. Make sure you know where you needed it in your solution.
- 6.11: One possible approach to this problem is to use the additional problem 1(b) from Part 1 and what you know about homomorphisms between groups.