

Math 817, Fall 2023, Dr. Honigs
Homework 3
Due Tues. **Oct. 17** at the start of class

Instructions:

- You are encouraged to work in groups. However, write your solutions in your own words.
- In your solutions, list any names of people you have worked with. (This is practice for listing collaborators!)
- In your solutions, refrain from using language like “clear”, “obvious”, or “easy”.

Homework 3 will be graded out of 50 points. There are 10 textbook questions, worth 5 points each.

Textbook questions:

Aluffi Ch. IV

- 1.12, 1.17
- 2.2, 2.6, 2.12, 2.14, 2.15, 2.16, 2.20, 2.21

Remarks:

- Here are some suggestions for deciding a group G is NOT simple:
 - Use Sylow III to analyze N_p the number of Sylow subgroups for each prime p dividing $|G|$. If $N_p = 1$ for some p , G isn't simple. (Why?) (cf. Example 2.13)
 - If Sylow III isn't enough to show $N_p = 1$, it may be possible to count numbers of elements in the group of different orders and force a contradiction if $N_p > 1$. (cf. Example 2.14)
 - Finally, the conjugation action of G on the Sylow p -subgroups induces a group homomorphism $G \rightarrow S_{N_p}$. It may be possible to conclude the kernel is proper and non-trivial. (cf. Example 2.15)
- You may use results from the section Ch. IV§2 even if we did not go over them in class (you will likely find them to be quite helpful).