

Math 817, Fall 2023, Dr. Honigs
Homework 4, Part 2
Due **Oct. 31** at the start of class

Instructions:

- You are encouraged to work in groups. However, write your solutions in your own words.
- In your solutions, list any names of people you have worked with. (This is practice for listing collaborators!)
- In your solutions, refrain from using language like “clear”, “obvious”, or “easy”.

Homework 4 Part 2 will be graded out of 50 points.

Questions 4.21, 5.12 and the additional question are each worth 10 points. Questions 4.20, 5.1, 5.7 and 5.9 are each worth 5 points.

Textbook questions:

Aluffi Ch. IV

- 4.20, 4.21
- 5.1, 5.7, 5.9, 5.12

Additional question:

1. Let N and H be groups and $\theta : H \rightarrow \text{Aut}_{\text{Grp}}(N)$ a group homomorphism. Let $N \rtimes_{\theta} H$ be the set $\{(n, h) \mid n \in N, h \in H\}$ with the operation

$$(n_1, h_1) \bullet_{\theta} (n_2, h_2) = (n_1 \theta_{h_1}(n_2), h_1 h_2).$$

Show that:

- (a) $N \rtimes_{\theta} H$ forms a group under the given operation.
- (b) Show there is a split short exact sequence $1 \rightarrow N \rightarrow N \rtimes_{\theta} H \rightarrow H \rightarrow 1$.