Math 817, Fall 2023, Dr. Honigs
Homework 6, Part 1
Due ~~**Nov. 28**~~ **Nov. 30** at the start of class

Instructions:

- You are encouraged to work in groups. However, write your solutions in your own words.
- In your solutions, list any names of people you have worked with. (This is practice for listing collaborators!)
- In your solutions, refrain from using language like "clear", "obvious", or "easy".

Homework 6 Part 1 will be graded out of 50 points. Each problem is worth 5 points.

Textbook questions:

Aluffi Ch. VIII

- 2.1, 2.7, 2.20, 3.2, 3.6, 3.9, 4.6, 4.7, 5.9, 5.15

Remarks:

- 2.1: The definition of a cyclic module was introduced way back in Ch. III. The point is just that $N = \langle n \rangle$ for some $n \in N$.
- 2.20: A module is flat if and only if tensoring an injection morphism with the module gives an injection. We didn't talk about applying a tensor to a homomorphism in class either, so sorting out what's going on here is part of doing this problem. You will need to use 2.19.
- You may ignore the paragraph starting "Deduce..." in 4.7.
- In 5.9 there are two induced maps $M \to M^\vee$ from a given bilinear pairing.
- 5.9 possible further hint: the main task here is to decide what this matrix is doing. Recall that if we give a matrix for a linear transformation, we must choose a basis for both the domain and codomain. Then the reference is helpful for concluding in the nondegenerate case. For showing A is nonsingular, one way to think about the determinant being a unit is to use the fact that in the nonsingular case the linear transformation will have an inverse.