

Math 818, Fall 2024, Dr. Honigs
Homework 2
Due Fri. Oct. 4

Instructions: You are encouraged to work in groups, but your final written solutions must be in your own words. At the top of your paper, write down the names of anyone you have worked with on the problem set.

Complete the following textbook exercises and questions.

Exercises:

- In Gathmann’s “Algebraic Geometry” (AG): 1.21, 2.18, 2.23, 2.36(a), 2.40, 12.15(a)

Hint for 12.15(a): You may find it useful to review the different Nullstellensatz statements in Ch. 10 of the CA notes.

Questions:

1. Let A be a ring. Recall that $\operatorname{Spec} A$ be the set of all prime ideals in A . Let I be an ideal of A . We define the subset $V(I) \subseteq \operatorname{Spec} A$ as follows:

$$V(I) = \{\mathfrak{p} \in \operatorname{Spec} A \mid I \subseteq \mathfrak{p}\}.$$

Verify that the set of all sets of the form $V(I)$ for some ideal I of A are the closed sets of a topology on $\operatorname{Spec}(A)$.

2. Given a set S with a topology \mathcal{T} , a *closed point* is an element $s \in S$ where $S \setminus \{s\} \in \mathcal{T}$.
 - (a) What are the closed points of $\mathbb{A}_{\mathbb{C}}^n$? What are the closed points of $\operatorname{Spec}(\mathbb{C}[x])$? Describe the closed points of $\operatorname{Spec}(A)$ for any ring A .
 - (b) What is the closure of $(y - x^2) \in \operatorname{Spec}(\mathbb{C}[x, y])$?
 - (c) What is the closure of $(0) \in \operatorname{Spec}(\mathbb{C}[x_1, \dots, x_n])$?