

Math 818, Fall 2024, Dr. Honigs
Homework 3
Due Fri. Oct. 18

Instructions: You are encouraged to work in groups, but your final written solutions must be in your own words. At the top of your paper, write down the names of anyone you have worked with on the problem set.

Complete the following textbook exercises and questions. (All are listed.)

Exercises:

- In Gathmann’s “Commutative Algebra” (CA): 6.13(a), 6.15
- In Gathmann’s “Algebraic Geometry” (AG): 4.13, 4.19

4.19: You can ignore the phrase “ringed spaces”. The question is asking about whether an isomorphism exists in the sense we defined in class.

Questions:

1. (a) Consider the ideal $I = (xy)$ in $k[x, y]$. Find $(I^e)^c$, where we are extending and contracting by the localization ring homomorphism $k[x, y] \rightarrow k[x, y]_{(x)}$.
(b) Use part (a) to prove that Proposition 6.7(c) of CA does not hold if we replace “prime ideal” with “ideal”.
2. Let R be a ring, I be an ideal in R and \mathfrak{p} be a prime ideal that contains I . Show the following result, which could be interpreted as saying that

$$(R/I)_{\mathfrak{p}} \simeq R_{\mathfrak{p}}/I_{\mathfrak{p}}.$$

(On the left-hand side \mathfrak{p} should be thought of as the corresponding ideal in R/I .) In fact, localization commutes with quotients more generally, but since we will use this particular fact in class, we will prove this case.