

Math 818, Fall 2024, Dr. Honigs
Homework 5
Due Fri. Nov. 22

Instructions: You are encouraged to work in groups, but your final written solutions must be in your own words. At the top of your paper, write down the names of anyone you have worked with on the problem set.

Complete the following textbook exercises and questions.

Exercises:

- In Gathmann’s “Algebraic Geometry” (AG): 7.28, 8.22, 9.18, 10.18
- In Gathmann’s “Algebraic Geometry” (CA): 11.21

11.21: Use the definition of dimension in terms of transcendence degree

Questions:

1. Let L/K be a field extension. Show that $z_1, \dots, z_d \in L$ are algebraically independent if and only if each extension $K(z_1, \dots, z_i)/K(z_1, \dots, z_{i-1})$ is transcendental.
2. Let X and Y be irreducible affine varieties. Show that a morphism $f : X \rightarrow Y$ is dominant if and only if $f^* : A(Y) \rightarrow A(X)$ is injective.

Hint: You may find it helpful to show that $f(X)$ is contained in a closed set $Z \subseteq Y$ if and only if $f^*(I(Z)) \subseteq I(X)$.

(Not to be turned in, just to consider: How would you adapt this argument for a dominant rational map $X \dashrightarrow Y$? A dominant rational map is equivalent to injectivity of $K(Y) \rightarrow K(X)$.)