Meet an SFU Mathematician: Dr. Katrina Honigs

Primitive Pythagorean triples

April 27, 2022

Today's talk:

- A bit about who I am and what my job is like
- Some math that I love

I work at Simon Fraser University



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- To get here, I completed a PhD degree, which focuses on an original research project.

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- Math is a very practical degree. Many jobs are available.
- My job at a university has a huge amount of freedom.
- There are many different areas of mathematical study involving different skills. Math has lots of different people.
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My research collaborators from a recent project:
Dr. Isabel Vogt
Dr. Sachi Hashimoto
Dr. Alicia Lamarche

- I am a “pure” mathematician: I work with proofs, which are careful explanations of why something must be true or false. Usually, proofs are an explanation of how something works.
- My area of math is algebraic geometry, which is about solutions to polynomial equations in more than one variable.

Question:

The whole numbers $3, 4, 5$ satisfy the equation $a^2 + b^2 = c^2$.
$5, 12, 13$ and $8, 15, 17$ do too:

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$
$$5^2 + 12^2 = 25 + 144 = 169 = 13^2$$
$$8^2 + 15^2 = 64 + 255 = 289 = 17^2$$

Can we find or describe how to find all triples of whole numbers that satisfy $a^2 + b^2 = c^2$?

Multiples of any solution are also solutions, like $6, 8, 10$, or $9, 12, 15$, etc.
We could also throw in negatives like $3, -4, 5$.

We'll look for positive whole number solutions that aren't multiples of other solutions, which are called primitive triples.

$3, 4, 5$ and $5, 12, 13$ and $8, 15, 17$ are all primitive triples.

Revised Question:
Can we describe the primitive triples satisfying $a^2 + b^2 = c^2$?

$a^2 + b^2 = c^2$ is the equation from the Pythagorean theorem.

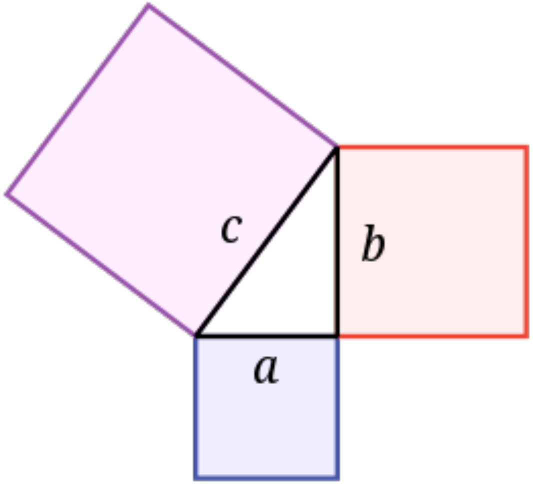


Illustration of statement of Pythagorean theorem

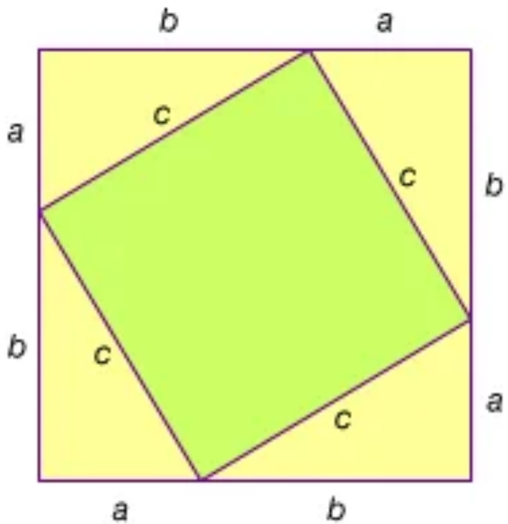


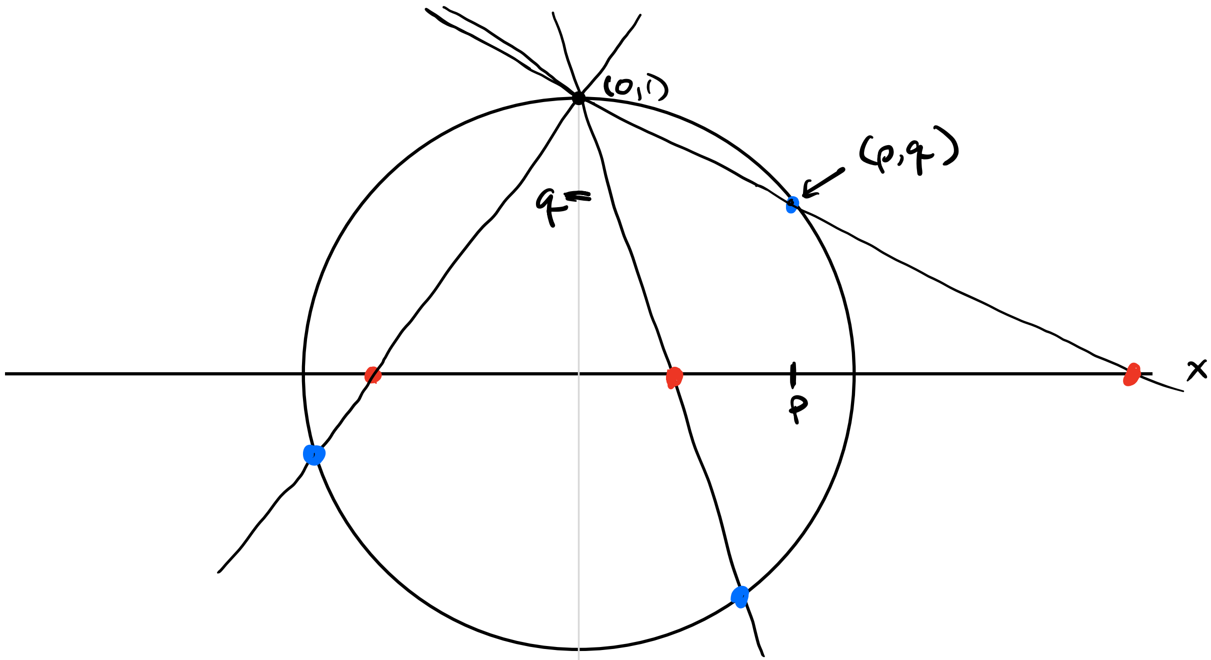
Illustration of why the Pythagorean theorem is true.

total area = yellow + green

$(a+b)^2 = a^2 + 2ab + b^2 = \underbrace{4 \cdot \tfrac{1}{2} \cdot ab}_{2ab} + c^2$

$a^2 + b^2 = c^2.$

A useful tool for us: Stereographic Projection



Stereographic projection gives a correspondence between points on the unit circle (except the “north pole”) and points on the x-axis.

If (p, q) is a point on the circle, where does it map to on the x-axis?

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We can take the line through (p, q) and $(0, 1)$:

$$y = \frac{q-1}{p}x + 1 \quad \text{or} \quad py = (q-1)x + p$$

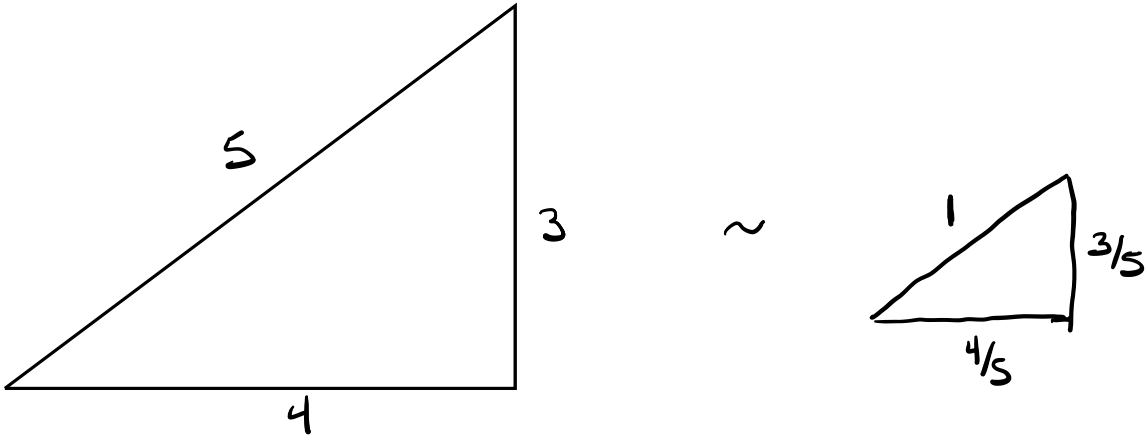
Set $y = 0$ to find the intersection with the x-axis:

$$0 = \frac{q-1}{p}x + 1$$

Stereographic projection maps (p, q) to $\frac{p}{1-q}$

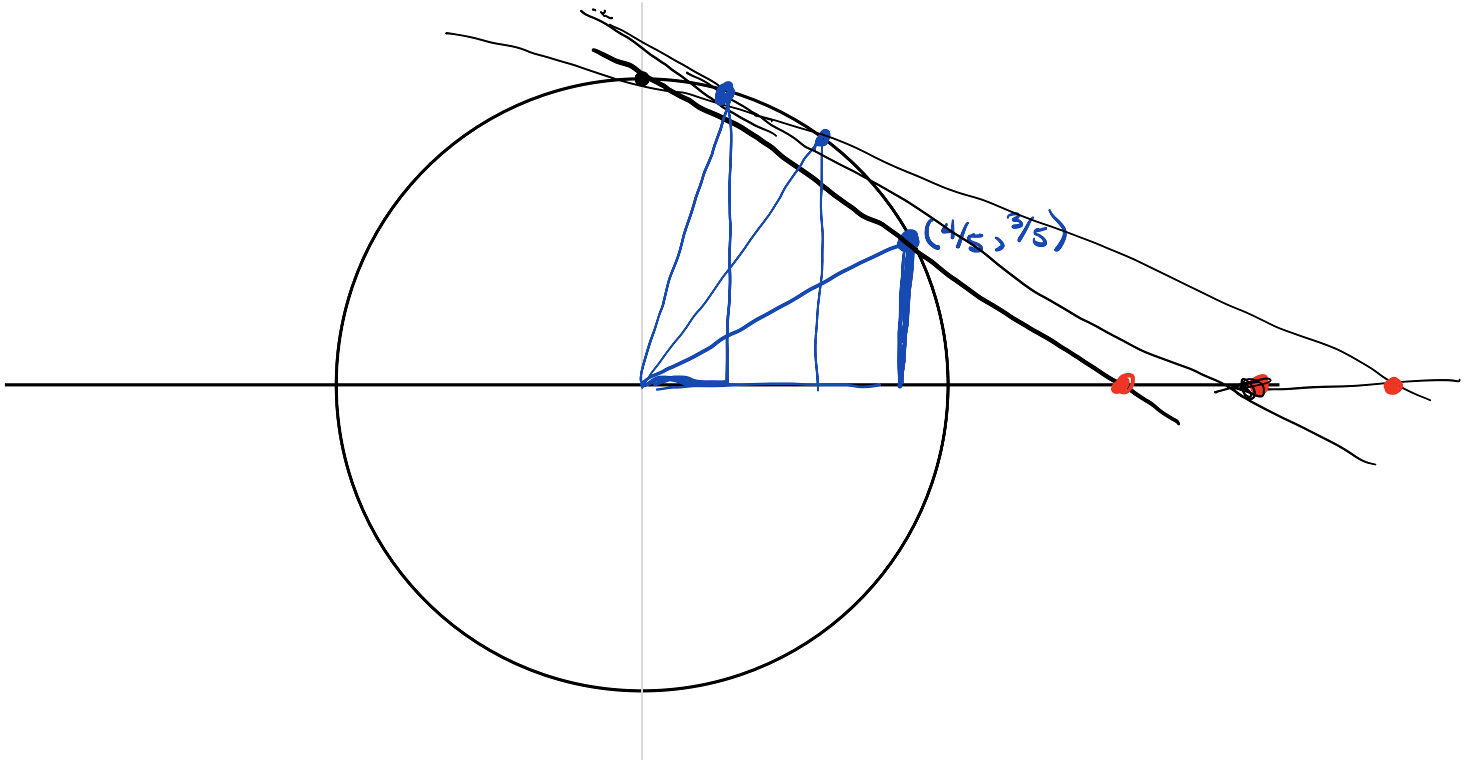
Triangles and Stereographic Projection

Triangles with whole number side lengths scale to triangles with a hypoteneuse of length 1 whose other side lengths are in $\mathbb{Q}$.



Triangles and Stereographic Projection

After scaling, triangles fit in the unit circle.

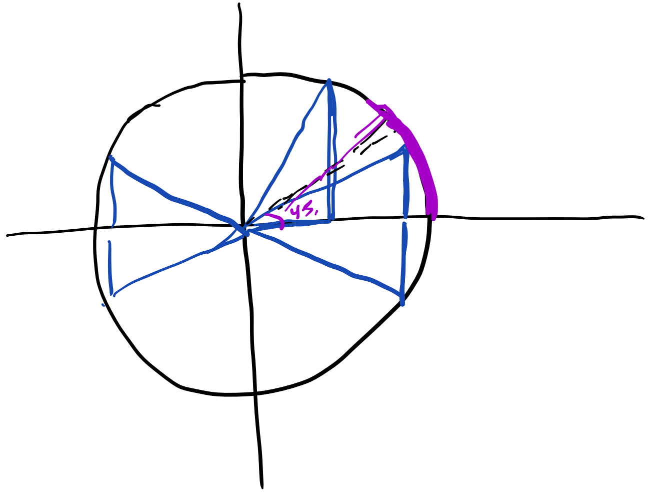


We showed stereographic projection maps (p, q) to $\frac{p}{1-q}$

$$\left(\tfrac{4}{5}, \tfrac{3}{5}\right) \qquad \frac{4/5}{1-3/5} = \frac{4/5}{2/5} = \frac{4}{2} = 2$$

We've shown every primitive Pythagorean triple corresponds to a point in $\mathbb{Q}$ on the x-axis!

Do all these points on the x-axis correspond to unique primitive Pythagorean triples?

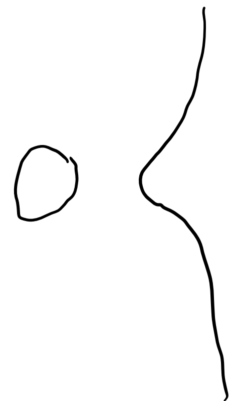


Fermat's Last Theorem

$$a^n + b^n = c^n \qquad n \geq 3$$

has "trivial" solutions with 0's, e.g. $0^n + 1^n = 1^n$

but no other whole # solutions



Proven in 1990's by Andrew Wiles

Related to elliptic curves

Advice for success as a university student:

- Read the comments you get on assignments
- Come to office hours or workshops for help!