

## Topics

- 1.) Dynamic Ricardian Model
- 2.) Transport Costs + Endogenously NT goods
- 3.) The Monetary Model of Nominal Ex. Rates

## Dynamic Ricardian Model

First, note that given our assumptions (identical, homothetic preferences, costless trade, etc), permanent shocks have no CA effects.

What about temporary productivity shocks?

Consider an initial symmetric equilibrium,

$$\bar{B}_0 = -\bar{\beta}_0^* = 0, \quad L = L^*, \quad A(L^*) = 1 \Rightarrow \bar{z}_0 = \frac{1}{2}$$

$$CA_t = B_{t+1} - B_t = \frac{w_t L}{P_t} + r_t B_t - c_t \quad \left. \begin{array}{l} \text{In units} \\ \text{of consumption} \\ \text{index} \end{array} \right\}$$

Suppose  $\alpha^* \downarrow$  across the board,  $\alpha^* = \frac{\alpha_0^*}{v} \quad v > 1$ ,  
for one-period, then reverts back to initial level

Assume steady-state with  $\beta(1+r) = 1$ ,

$$\bar{c} = \bar{r} \bar{B} + \frac{\bar{w} L}{\bar{P}}$$

$$\hat{\bar{c}} = \frac{\bar{r} d \bar{B}}{\bar{c}_n}$$

Log-differentiate the Euler Eq.,

$$\hat{\bar{c}} = (1-\beta) \hat{r} + \hat{c}$$

Since productivity returns to its initial level,

$$\hat{\bar{c}} + \hat{\bar{c}}^* = 0$$

Add Euler eqs., and use this to get,

$$\hat{r} = -\frac{1}{1-\beta} \left( \frac{\hat{c} + \hat{c}^*}{2} \right)$$

Next, note that

$$P(c + c^*) = wL + w^*L \quad \begin{matrix} \text{log-diff} \\ \text{and use symmetry} \end{matrix}$$

$$\Rightarrow \frac{\hat{c} + \hat{c}^*}{2} = \frac{\hat{w} + \hat{w}^*}{2} - \hat{p}$$

Log-diff the consumption-based price index,

$$\hat{p} = \frac{\hat{w} + \hat{w}^*}{2} - \frac{\hat{v}}{2} \Rightarrow \frac{\hat{c} + \hat{c}^*}{2} = \frac{\hat{v}}{2}$$

Therefore,

$$\hat{r} = \frac{-\hat{v}}{2(1-\beta)} < 0$$

} Both countries want to save, so  $r$  must fall

Finally, log-diff. the Natl. Income identity,

$$\frac{d\bar{B}}{\bar{c}_0} = \hat{w} - \hat{p} - \hat{c} = \frac{\hat{w} - \hat{w}^* + \hat{v}}{2} - \hat{c}$$

Since  $w/w^* = \frac{A(\bar{z})}{v}$

$$\hat{w} - \hat{w}^* = -\hat{v} + A'(\bar{z}) d\bar{z}$$

Using the  $B(\cdot)$  curve + symmetry,

$$\hat{w} - \hat{w}^* = 4 d\bar{z}$$

Therefore,

$$\hat{w} - \hat{w}^* = \frac{-\hat{v}}{1 - \bar{v} A'(\bar{z})}$$

Use this and the Euler eq. to sub-out  $\hat{c}$

$$\hat{c} = \hat{c} - (1-\beta) \hat{r} = \frac{\bar{r} d\bar{B}}{\bar{c}_0} - (1-\beta) \left[ \frac{-\hat{v}}{2(1-\beta)} \right]$$

$$\frac{d\bar{B}}{\bar{c}_0} = \frac{-\hat{v}}{(1+\bar{r}) [2 - \bar{v} A'(\bar{z})]} < 0$$

} Home runs  
a current deficit  
and accumulate  
foreign debt

H's first period real income rises, but falls starting in second period. First period gain dominates. Envelope

## Transport Costs + Non-Traded Goods

Suppose  $k < 1$  units "melt" during transit.

What are the implications for current accounts, terms of trade, and the real ex. rate?

This is a very active research topic these days.

Let's first consider the implications for trade.

1.) Note, it costs  $H w_a$  to produce a unit, but it costs  $\frac{w^* a^*}{1-k}$  to import a unit.

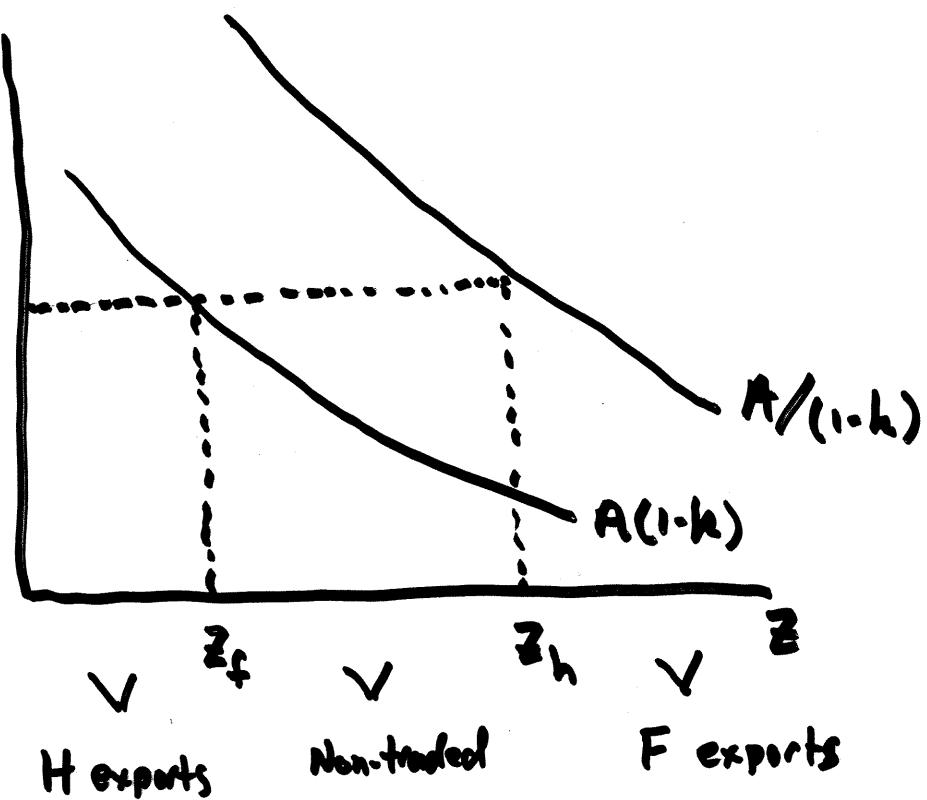
$\Rightarrow H$  produces as long as  $w_a < \frac{w^* a^*}{1-k}$

or 
$$\boxed{\frac{w}{w^*} < \frac{A}{1-k}}$$

2.) F produces goods as long as  $w^* a^* < \frac{w_a}{1-k}$

or 
$$\boxed{\frac{w}{w^*} > (1-k) A}$$

Graphically,



Note,

$$P = \exp \left\{ \int_0^{z_h} \log(wa(z)) dz + \int_{z_h}^1 \log \frac{w^* a^*(z)}{1-h} dz \right\}$$

$$P^* = \exp \left\{ \int_0^{z_f} \log \frac{wa(z)}{1-h} dz + \int_{z_f}^1 \log w^* a^*(z) dz \right\}$$

Therefore, the Real Ex. Rates becomes,

$$\frac{P}{P^*} = \exp \left\{ \int_{z_f}^{z_h} \log \frac{wa}{w^* a^*} dz + [z_f - (1-z_h)] \log(1-h) \right\}$$

Real Ex. Rate depends on : 1.) Rel. Price of NT goods  
2.) Shipping costs + Intl. pattern of production

What about the B(.) schedule?

$$PC + P^*C^* = wL + w^*L \quad \left. \begin{array}{l} \text{Note, price indexes} \\ \text{include transport costs} \end{array} \right\}$$

$$\begin{aligned} wL &= z_h PC + z_f P^*C^* \\ &= z_h PC + z_f (wL + w^*L - PC) \end{aligned}$$

$$\text{Note, } TB = wL - PC$$

Therefore,

$$\frac{w}{w^*} = \frac{L^*/L}{1-z_h} \left\{ z_f - \frac{(z_h - z_f)TB}{L^*/a^*(1)} \right\}$$

(using the fact that  $w^* = P^*(1)/a^*(1) = 1/a^*(1)$ )

Note, we can use the following relationship to sub out  $z_h$  in terms of  $z_f$ .

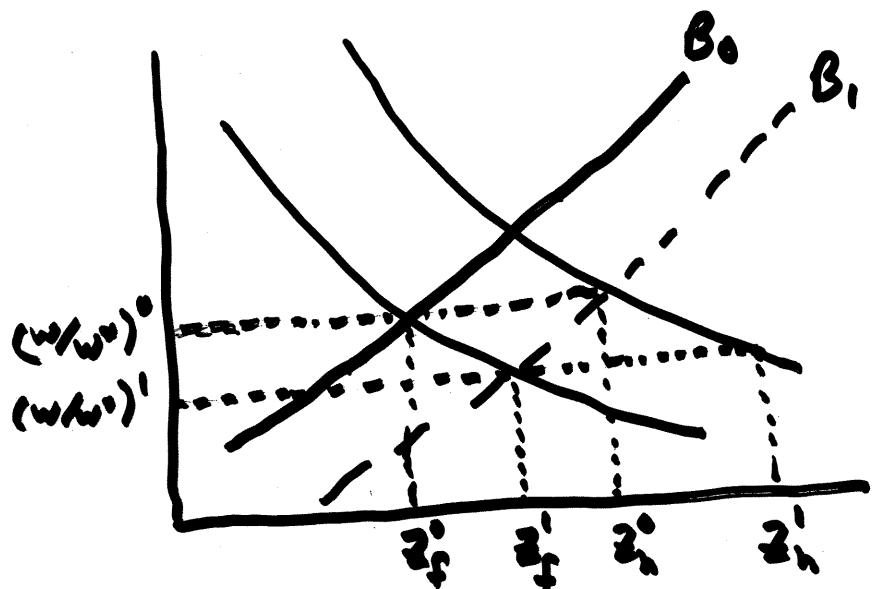
$$(1-k) A(z_f) = A(z_h)/(1-k)$$

For example, suppose  $A(z) = \exp(1-2z)$

$$z_h = z_f - \log(1-k)$$

Suppose  $TB \uparrow$  exogenously } if H makes positive  
resource transfer to F

Note, this shifts down the  $B_C$  schedule.



} Note: Now the int'l. dist. of wealth matters!

- 1.) H's relative wage declines
- 2.) H's TOT  $\downarrow$
- 3.) H's Real Ex. Rate depreciates
- 4.) H's exports expand, imports contract

### Intuition

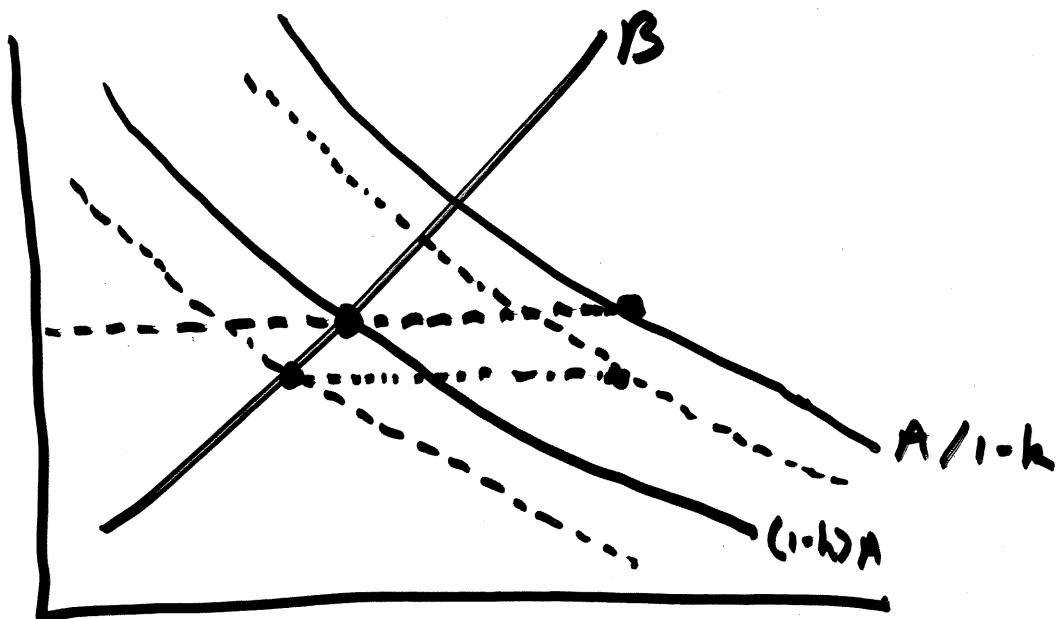
F spends part of the transfer on NT goods  
This draws F labor into NT goods, and reduces  
output in F's export sector.  $\rightarrow$  Relative prices  
of F's NT + T goods rise

What about productivity shocks?

Two cases:

1.) Permanent.  $a^* \downarrow$  across the board, forever.

If initially  $TB = 0$ , then  $TB$  continues  $\approx 0$ .



1.) H's relative wage falls

2.) H's real wage and TOT  $\uparrow$

3.) H's Real Ex. Rate  $\downarrow$  ( $P/P^*$ )  $\downarrow$

4.)  $Z_f \downarrow$  (H exports  $\downarrow$ )  $Z_L \downarrow$  (F's exports  $\uparrow$ )

Note: Now TOT + RER move in opposite directions.

2.) Temporary. Suppose  $a^* \downarrow$  for only one period.

Now even temporary shocks have long lasting effects, since the presence of NT goods produces a "home market effect" [Keynes-Olin transfer problem debate].

### Summary of main effects

- 1.) During the period of higher foreign productivity, H runs CA def. as before.
- 2.) Also, H experiences a short-run TOT and RER appreciation
- 3.) In the long-run, H makes a positive transfer to F (servicing its foreign debt).
- 4.) Since B schedule is now lower, H's relative wages are permanently lower
- 5.) As a result, H's export sector expands permanently.
- 6.) On net, H's initial gain surpasses its subsequent losses.

## Exchange Rate Dynamics

Until now, our models have been entirely real.  
(No money, only relative prices).

Now we introduce money, and the distinction between nominal and real exchange rates.

During the 1970s + early 1980s, models of exchange rate determination were actively studied.

Work by Meese & Rogoff (1983) led to a warning of interest in ex. rate models.  
They showed : 1.) Traditional ex. rate models could not forecast out-of-sample better than a naive random walk model  
2.) Traditional models could not even explain ex. rate changes ex post.

Recently, there has been a revival of interest in exchange rate modeling, triggered by several influential papers,

1.) Mark (AER, 1995)

- Long-horizon predictability

2.) Engel + West (JPE, 2005; NBER Macro Annual/2008)

- Random walk ex. rates as discount factor  $\uparrow 1.0$

3.) Gourinchas + Rey (JPE, 2007)

- Valuation Effects, Revival of port. balance models

4.) Bacchetta + van Wincoop (AER, 2006)

- Imperfect Common Knowledge + Higher-Order Beliefs

# The Monetary Model of Exchange Rates

## 3 ingredients

1.) Reduced form money demand + Exog. monetary policy

$$\cdot m_t - p_t = \alpha y_t - \eta i_t$$

$$m_t^* - p_t^* = \alpha y_t^* - \eta i_t^*$$

2.) PPP / Flexible Prices

$$\cdot P_t = s_t + P_t^*$$

3.) Uncovered Interest Parity (Risk-Neutrality)

$$\cdot i_t^* = i_t + E_t(s_{t+1} - s_t)$$

These 3 ingredients can be interpreted as expressing equil. in the money market, the goods market, and the bond market.

Note: This is a partial equilibrium model

Although each of these 3 ingredients rests on shaky empirical ground, let's begin by studying their implications.

Differencing money demands yields,

$$P_t - P_t^* = m_t - m_t^* - \alpha(y_t - y_t^*) + \gamma(i_t - i_t^*)$$

Using PPP + UIP we can write this as,

$$S_t = f_t + \gamma [E_t S_{t+1} - S_t]$$

where  $f_t = m_t - m_t^* - \alpha(y_t - y_t^*)$  defines the so-called monetary model fundamentals.

We can write this as,

$$S_t = (1 - \beta) f_t + \beta E_t S_{t+1}$$

$$\beta = \frac{\gamma}{1+\gamma}$$

so that  $S_t$  is a convex combo. of current fundamentals and next period's expected ex. rate.

What is a reasonable value for  $\beta$ ?

$$\eta = \left| \frac{dm}{di} \right| \frac{1}{m} \Rightarrow i \cdot \eta = \text{interest elast. of money demand}$$

$$\approx (.3 - .5)$$

$$\Rightarrow \eta \approx (30, 50) \text{ when } i = .01 \text{ (quarterly data)}$$

$\rightarrow \beta > .95 \rightarrow \text{Ex. Rate mainly determined by expectations!}$

$$S_t = (1-\beta)f_t + \beta E_t S_{t+1}$$

Iterate forward, using "law of iterated expectations" [caveat: Heterogeneous Expectations!]

$$S_t = (1-\beta)E_t \sum_{j=0}^T \beta^j f_{t+j} + \beta^{T+1} E_t S_{t+T+1}$$

$$= (1-\beta)E_t \sum_{j=0}^{\infty} \beta^j f_{t+j} \quad \text{if } \lim_{T \rightarrow \infty} \beta^T E_t S_{t+T} = 0$$

\ No Bubbles Condition

### Example

Suppose  $f_t = \rho f_{t-1} + \varepsilon_t$ ,  $|\rho| < 1$ . Then,

$$S_t = \left( \frac{1-\rho}{1-\rho\rho} \right) f_t. \quad \text{This implies,}$$

$$\text{var}(S_t) = \left( \frac{1-\rho}{1-\rho\rho} \right)^2 \text{var}(f_t) < \text{var}(f_t) \quad [\rho\rho < \rho]$$

More generally, define

$$PV_t = \sum_{j=0}^{\infty} \beta^j f_{t+j}$$

Using this we have the orthogonal decomposition,

$$PV_t = E_t(PV_t) + U_t$$

$$\Rightarrow \text{Var}(PV_t) = \text{Var}(E_t PV_t) + \text{Var}(U_t)$$

Applying this to our ex. rate equation,

$$\text{var}(s_t) = \text{var}(E_t P V) < \text{var}(P V) \quad \left. \begin{array}{l} \text{Shiller} \\ \text{Bound} \end{array} \right\}$$

That is, ex. rates should be "smoother" than their ex post realized future fundamentals.

This inequality is strongly rejected by the data.

One response is to consider the possibility of bubbles. Note, with fiat currency, standard TVC doesn't apply to rule out explosive bubbles.

$$s_t = s_t^f + \beta_t, \text{ where } \beta_{t+1} = \frac{1}{\rho} \beta_t + \varepsilon_{t+1}$$

$\begin{matrix} \checkmark \\ \text{fundamentals} \end{matrix} \quad \begin{matrix} \checkmark \\ \text{Bubble} \\ \text{solution} \end{matrix}$

Bubbles usually ruled out on empirical grounds, since a bubble path is explosive.

However, consider the following process,

$$\beta_{t+1} = \frac{1}{\rho \pi} \beta_t + \varepsilon_{t+1} \quad \text{w.p. } \pi \quad \left. \begin{array}{l} \text{"collapsing} \\ \text{Bubble} \end{array} \right\}$$
$$= \varepsilon_{t+1} \quad \text{w.p. } (1-\pi) \quad \left. \begin{array}{l} \text{Blanchard (1979)} \end{array} \right\}$$

Note that  $E_t \beta_{t+1} = \frac{1}{\rho} \beta_t$

# Testing for Bubbles [Hausman(1978), West(1987) Meese(1986)]

Basic idea : Compare estimates from Euler eq. and PV model. Should be the same under the null of no bubbles. [PV model imposes no bubbles const., Euler eq. doesn't]

## Euler

$$E_t S_{t+1} = V \beta_1 S_t - \frac{1-\beta_1}{\beta_1} f_t$$

orth.  
forecast error

RE orth. decomposition  $S_{t+1} = E_t S_{t+1} + V_{t+1}$

This gives,

$$S_{t+1} = V \beta_1 S_t - \frac{1-\beta_1}{\beta_1} f_t + V_{t+1}$$

## PV

$$S_t = \frac{1-\beta_2}{1-\beta_1 \beta_2} f_t + \text{"Shiller Error"}$$

info used to forecast  $f_t$  not contained in history of  $f_t$

$$H_0: \beta_1 = \beta_2 \quad (\text{No Bubbles})$$

$$H_1: \beta_1 \neq \beta_2 \quad (\text{Bubbles})$$

## Mixed Results

Basic Problem :  $\hat{\beta}_1 \neq \hat{\beta}_2$   
might differ for other reasons, e.g., model misspecification.