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Nontraded goods, nontraded factors, and international non-diversification

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Abstract

Can the presence of nontraded consumption goods or nontraded factors of production explain the high degree of "home bias" displayed by investor portfolios? We find that the answer is no, so long as individuals have access to free international trade in financial assets. In particular, it is never optimal to exhibit home bias with respect to domestic traded-good equities. By contrast, holdings of nontraded-good equities in an optimal portfolio will depend sensitively on the elasticity of substitution between traded and nontraded goods.

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1. Introduction

A basic lesson from finance is that it is optimal to diversify one's portfolio. It is consequently surprising that there is substantial "home bias" in observed portfolios, in the sense that investors hold a disproportionate share of their portfolios in the form of domestic assets. Many researchers have suggested that the explanation for home bias is related to the fact that a large fraction of consumption

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goods are not traded internationally.[1] Because domestic consumers must purchase the entire endowment of domestic nontraded goods, it may be desirable for them to allocate a large fraction of their portfolio to the domestic nontraded good equity which has high payouts when the output of the nontraded good industry is high. However, we will show that the presence of nontraded goods cannot resolve the home bias puzzle in a world with frictionless trade in financial assets.

Our framework is a multi-country general equilibrium model with complete security markets. Individuals in each country value two consumption goods, one traded and one nontraded, which enter nonseparably in the individual's utility function. Endowments of both types of goods are stochastic. While the traded good may be transported costlessly across countries, residents of each country must consume the entire endowment of the nontraded good. Frictionless trade in financial assets means that individuals in all countries are free to trade equities whose payouts depend on endowments of both traded goods and nontraded goods.

We take an indirect route to characterizing optimal portfolio holdings. First, we characterize optimal consumption allocations. We then determine portfolio holdings that could support these optimal allocations in a decentralized market setting. The remainder of the paper is therefore structured as follows. Section 2 characterizes optimal consumption allocations for each country in the world economy. Because there is only one traded good, the problem amounts to specifying the optimal amount of the traded good that each country should consume conditional on its endowment of the nontraded good. For example, if the traded and nontraded goods are complements in consumption, it is optimal to allocate more of the traded good to countries with high endowments of the nontraded good. In general equilibrium, however, the world resource constraint for the traded good means that, even if all countries receive a relatively high endowment of the nontraded good, not all countries can have relatively high consumption of the traded good. Rather, a country will receive a relatively high allocation of the traded good only if its endowment of the nontraded good is high relative to the world average endowment of the nontraded good. Specifically, we show that a country's optimal allocation of the traded good depends on three factors: the world endowment of the traded good; the world endowment of the nontraded good; and the country's individual endowment of the nontraded good. This finding suggests a ‘‘three-fund theorem’’ – i.e., it suggests that a three-fund portfolio strategy can support the optimum.

Section 3 describes portfolio holdings that can support the optimal allocations in a decentralized economy. In doing this, we exploit the fact that the optimum derived in Section 2 can be supported as a competitive equilibrium with

[1] Early investigations by Krugman (1981) and Stulz (1983) studied how a preference bias for domestic goods affects optimal portfolio composition. Subsequently, the contributions of Eldor et al. (1988); Stockman and Dellas (1989); Tesar (1993, 1994) and Pesenti and van Wincoop (1994) studied how explicit incorporation of nontraded goods influences portfolio choice.

appropriate allocations of wealth when the planner's shadow prices are reinterpreted as equilibrium prices in competitive markets. As noted above, the optimal allocations depend on three factors: the world endowment of the traded good; the world endowment of the nontraded good; and the country's individual endowment of the nontraded good. If individuals could trade claims on these three factors and if the shocks are small, then three ''basic securities'' can be used to construct a portfolio that would pay individuals just enough to purchase their optimal consumption bundles. However, assets traded on real-world market exchanges have payouts denominated in units of a common numeraire – dollars, for example. In this setting, a claim on the output of a particular industry has a payoff that depends both on the realization of the endowment process for that industry, but also depends on the realization of the relative price of that industry's output in terms of the numeraire. We discuss how to construct and value ''market securities'' that pay off in units of a common numeraire which we take to be the traded good. Further, we describe how to construct the basic securities as linear combinations of the market securities. This section also discusses the relationship between our results and those obtained in the earlier literature. Section 4 shows how the optimal portfolio changes in the presence of risk associated with nontraded human capital. Section 5 concludes.

2. Optimal allocations

We use a multi-country general equilibrium model with complete financial markets to study the determinants of optimal portfolio choice in the presence of nontraded consumption goods. We begin by characterizing optimal allocations of traded and nontraded goods; the next section shows how to structure financial portfolios to support these optimal allocations.

There are J countries, indexed by $j = 1,2,...,J$. Within each country, the tradable and nontradable consumption goods arrive as non-produced endowments. The per capita endowment of the traded good in country j is denoted by ξ_j and the per capita endowment of the nontraded good in country j is denoted by ζ_j. Individuals value consumption of both the tradable and nontradable goods; we let x_j denote country j's per capita consumption of the tradable good, and let z_j denote country j's per capita consumption of the nontradable good.

Pareto optimal allocations maximize a weighted sum of individual country utilities $v(x_j, z_j)$. Letting ω_j denote the weight given to country j with $\Sigma_{j=1}^{J} \omega_j = 1$, this weighted sum is given by:

$$\sum_{j=1}^{J} \omega_j v(x_j, z_j)$$

The resource constraint for the traded good is:

$$\sum_{j=1}^{J} \pi_j x_j = \sum_{j=1}^{J} \pi_j \xi_j \equiv X \tag{1}$$

where X denotes the world endowment of the traded good. The resource weights π_j allow countries to vary in terms of economic size. We use λ to denote the shadow price (Lagrange multiplier) associated with the resource constraint (Eq. (1)). This shadow price reflects the marginal utility of an additional unit of the traded good, which will be equated across countries in an optimal allocation scheme.

The first-order conditions describing optimal allocations of x_j are

$$\omega_j \frac{\partial v(x_j, z_j)}{\partial x_j} = \lambda \pi_j, \quad j = 1, 2, \ldots, J \tag{2}$$

together with the resource constraint, (Eq. (1)). Eq. (2) implies that optimal allocations are of the form

$$x_j = \Psi_j(X, \{z_j\}_{j=1}^{J}, \{\pi_j\}_{j=1}^{J}, \{\omega_j\}_{j=1}^{J})$$

In particular, optimal allocations depend on the world quantities of the traded good and on all of the individual country quantities of nontraded goods. To learn about the nature of this dependence, it is useful to adopt the following two-step strategy. First, we study the properties of the ''Frisch demand curve'' for x_j as a function of λ, where z_j is interpretable as a shift variable in the demand curve (see Frisch (1932)). That is: we will study the properties of

$$x_j = \psi\left(\lambda, z_j; \frac{\omega_j}{\pi}\right) \tag{3}$$

as implicitly defined by (Eq. (2)).

Second, given (Eq. (3)), the equilibrium value of λ is determined by the world resource constraint for the traded good:

$$\sum_{j=1}^{J} \pi_j \psi\left(\lambda, z_j; \frac{\omega_j}{\pi_j}\right) = X \tag{4}$$

This two-stage procedure has the additional benefit that we explicitly solve for the behaviour of λ, which is the world shadow price of an additional unit of the traded good. This shadow price is important in our subsequent analysis of market equilibrium.

2.1. Restrictions on preferences

To characterize the optimum in detail and to interpret it, we specialize the utility function to the following:

$$v(x, z) = \left(\frac{1}{1-\sigma}\right)\Phi(x, z)^{1-\sigma}$$

where the aggregator function Φ is homogeneous of degree one in x and z. For simplicity, we have suppressed the "j" subscript that indexes countries.

In this specification, the parameter σ is interpretable as relative risk aversion with respect to the composite good $c \equiv \Phi(x, z)$; σ is also the reciprocal of the elasticity of substitution across time or across states of nature.

Since the aggregator Φ is homogeneous of degree one, it has two properties that we use repeatedly below. First, we define $\phi(x/z) \equiv \Phi((x/z), 1)$ and then express momentary utility as:

$$v(x, z) = \left(\frac{1}{1-\sigma}\right)z^{1-\sigma}\phi(x/z)^{1-\sigma}$$

Second, the local behaviour of this function can be approximated by a constant-elasticity-of-substitution function. The marginal rate of substitution between x and z is $m = [\partial\Phi/\partial x]/[\partial\Phi/\partial z]$, and its elasticity with respect to (x/z), which we denote by $-\mu$, is given by

$$-\mu \equiv \frac{\mathrm{d}m}{m} \div \frac{\mathrm{d}(x/z)}{(x/z)}.$$

When goods are allocated optimally, the shadow relative price of the nontraded good in terms of the traded good is given by

$$p \equiv \frac{1}{m},$$

so that

$$\frac{\mathrm{d}p}{p} = \mu\frac{\mathrm{d}(x/z)}{(x/z)} \tag{5}$$

The elasticity of substitution between x and z, $1/\mu$, is then given by

$$\frac{1}{\mu} = \left[\frac{\mathrm{d}(x/z)}{(x/z)} \div \frac{\mathrm{d}p}{p}\right]$$

Finally, the shares of x and z in total consumption expenditure are

$$s_x = \frac{x}{\Phi}\frac{\partial\Phi}{\partial x};\; s_z = \frac{z}{\Phi}\frac{\partial\Phi}{\partial z}$$

2.2. Properties of optimal allocations: step 1

We now proceed to determine the properties of optimal allocations as revealed by the ψ function. Totally differentiating Eq. (3), we find that

$$\frac{\mathrm{d}x}{x} = \left[\frac{1}{s_z\mu + \sigma s_x}\right]\frac{\mathrm{d}\lambda}{\lambda} + \left[\frac{(\mu - \sigma)s_z}{s_z\mu + \sigma s_x}\right]\frac{\mathrm{d}z}{z} \tag{6}$$

where we continue to suppress country subscripts since we are describing general properties of the ψ function. Note that (Eq. (6)) determines the elasticities of the ψ function:

$$\eta^{\lambda} \equiv -\left[\frac{1}{s_z\mu + \sigma s_x}\right]$$

$$\eta^{z} \equiv \left[\frac{(\mu - \sigma)s_z}{s_z\mu + \sigma s_x}\right]$$

Eq. (6) tells us whether an increase in z raises or lowers the optimal level of x, holding fixed the shadow price λ. If the substitution of x for z within the consumption aggregator is small (i.e., if μ is large relative to σ), then marginal utility of x increases with z, and it is optimal to allocate larger quantities of the traded good to countries with higher endowments of the nontraded good. Conversely, if μ is small relative to σ so that the marginal utility of x decreases with z, then the optimal allocation involves lower provision of the traded good to countries experiencing relatively high endowments of the nontraded good. If the function ψ is interpreted as a Frisch (1932) demand curve for x, then an increase in z is a positive demand shifter for the traded good if $\mu - \sigma > 0$, and a negative demand shifter if $\mu - \sigma < 0$.[2]

2.3. Properties of optimal allocations: Step 2

The Frisch demand curves (Eq. (3)) together with the resource constraint (Eq. (4)) can be used to describe general properties of Pareto optimal allocations. Eq. (3) tells us that any country's allocation of the traded good only depends on the world shadow price λ, on the country's own nontraded good quantity, $z_j = \zeta_j$, and on the ratio ω_j/π_j. In turn, the world resource constraint (Eq. (4)) for the traded good tells us that the shadow price depends on the world endowment of the traded good and on the vectors of nontraded good quantities. Even with more sources of heterogeneity, optimal allocations will never depend on individual country realizations of the traded good endowment $\{\xi_j,\ j = 1, 2, \ldots, J\}$.

To characterize the optimal allocations more explicitly, we totally differentiate the resource constraint (Eq. (1)). Defining $\theta_j = \pi_j x_j/X$, $\eta^Z \equiv \Sigma_{j=1}^{J}\ \theta_j\eta_j^z$ and $\mathrm{d}Z = [\Sigma_{j=1}^{J}\ \theta_j\eta_j^z\ (\mathrm{d}z_j/z_j)]/\eta^Z$ and rearranging terms, we arrive at an expression that

[2] A similar line of reasoning concerning the relative importance of μ versus σ can be found in Tesar (1993, 1994) and Pesenti and van Wincoop (1994).

determines the effect on λ of changes in the endowments of the traded and nontraded goods:

$$\frac{d\lambda}{\lambda}=\left[\frac{1}{\sum_{j=1}^{J}\theta_j\eta_j^{\lambda}}\right]\frac{dX}{X}-\left[\frac{\eta_Z}{\sum_{j=1}^{J}\theta_j\eta_j^{\lambda}}\right]\frac{dZ}{Z} \tag{7}$$

From Eq. (7) we have the intuitive result that the world marginal utility of X falls with increases in X since $\Sigma_{j=1}^{J}\ \theta_j\eta_j^{\lambda}<0$. Further, we can view λ as influenced by changes in the world quantity of the traded good, dX/X, and by changes in the world ''demand'' for (allocations of) traded goods, as related to dZ/Z.

Using (Eq. (7)) together with Eq. (6) we find that, for country j,

$$\frac{dx_j}{x_j}=\frac{\eta_j^{\lambda}}{\sum_{j=1}^{J}\theta_j\eta_j^{\lambda}}\frac{dX}{X}-\frac{\eta_j^{\lambda}}{\sum_{j=1}^{j}\theta_j\eta_j^{\lambda}}\eta^{Z}\frac{dZ}{Z}+\eta_j^{z}\frac{dz_j}{z_j} \tag{8}$$

where the first two terms on the right-hand-side of (Eq. (8)) reflect the effects of aggregate displacements operating through λ. This equation shows that there is a positive effect of world traded good supply (dX/X) on the allocation of x to country j and that there are also effects of the country's nontraded good consumption (dz_j/z_j) and that of the rest of the world (dZ/Z).

It is possible to establish quite general propositions concerning properties of Pareto optimal allocations and their supporting portfolios. However, it is simpler and more intuitive to study a ''symmetric'' world economy in which all countries are identical in terms of initial conditions expressed in per capita terms, although they are subject to different shocks to their endowments of traded and nontraded goods. That is: we assume the following initial conditions: $x_j=x=X$, $z_j=z$, $p_j=p$, and $\pi_j=\omega_j$ for each country $j=1, 2, \ldots, J$. With identical preferences across countries, the key elasticities are the same across countries: $\eta_j^{\lambda}=\eta^{\lambda}$, $\eta_j^{z}=\eta^{z}=\eta^{Z}$, $\mu_j=\mu$.[3] However, even with ex-ante symmetry, optimal allocations will still differ across countries because of variation across countries in endowments of nontraded goods. Below, variables that differ across countries continue to be distinguished by the subscript j.

Under symmetry, Eq. (8) simplifies to the following:

$$\frac{dx_j}{x}=\frac{dX}{X}+\eta^{Z}\left(\frac{dz_j}{z}-\frac{dZ}{Z}\right) \tag{9}$$

where $dZ/Z\equiv\Sigma_{j=1}^{J}\ \theta_j(dz_j/z_j)$.

[3] In general, these elasticities may differ across countries because the ratio π/ω differs across countries or because the benchmark level of z at which the elasticities are evaluated differs across countries. The symmetry assumption simplifies the algebra greatly. See our working paper, Baxter et al. (1995), for analysis of the general case.

As Eq. (9) illustrates, changes in world supplies of X are shared equally if $\eta^Z=0$. If, however, $\eta^Z\neq 0$, then an additional reallocation of the traded good is undertaken based on an individual country's endowment of the nontraded good relative to the world average. Thinking about the nontraded good as producing shifts in the demand for the traded good, equation (Eq. 9) is very intuitive: changes in world demand for x must be frustrated by adjustments in its shadow price (λ) since there is an exogenously given stock to be allocated. It is only if there is a relative demand shift that a country's allocation is affected.

3. Analysis of supporting portfolios

The preceding section provided a characterization of Pareto optimal allocations in a multi-country model with endowments of traded and nontraded goods. We found that each country should consume its endowment of the nontraded good and that the allocation of the traded good would take the general form

$$x_j = \Psi_j(X, \{z_j\}_{j=1}^{J}, \{\pi_j\}_{j=1}^{J}, \{\omega_j\}_{j=1}^{J})$$

If we now view the endowments as stochastic, then the optimal allocations corresponding to a specific choice of welfare weights ($\{\omega_j\}_{j=1}^{J}$) and population weights ($\{\pi_j\}_{j=1}^{J}$) can be realized as an outcome to an ex-ante market equilibrium with complete contingent claims as in the traditional general equilibrium theory of Arrow (1964) and Debreu (1959).[4] However, it is not always the case that one needs a full menu of Arrow-Debreu securities to support Pareto optimal allocations. In some circumstances, it is sufficient to use a smaller number of securities to achieve the optimal allocations.

Our objective in this section of the paper is to describe such portfolios. Accordingly, we restrict attention to shocks that are sufficiently small that we can assume that two key classes of functions are approximately linear: these are (i) the functions describing the Pareto optimal allocations and (ii) the functions describing the payouts from the ''market securities'' that we describe in the remainder of this section. We find that optimal allocations for country j can be supported by holding appropriate quantities of three ''mutual funds:'' (i) a portfolio that is a claim to the world's traded-good endowment; (ii) a portfolio that is a claim to the world's nontraded-good endowment (suitably defined); and (iii) a claim to the endowment of country j's nontraded good. Consistent with our discussion of the general properties of Pareto optimal allocations, we find that there is no separate

[4]These contingent claims deliver one unit of the traded good if the state of nature is a specific realization of the vector (X, $\{z_j\}_{j=1}^{J}$), and zero otherwise. The specific allocation corresponding to ($\{\pi_j\}_{j=1}^{J}$,$\{\omega_j\}_{j=1}^{J}$) also typically requires a transfer of wealth across countries unless there are specific values for the welfare weights ($\{\omega_j\}_{j=1}^{J}$).

role for claims on country j's traded-good output – holding such an asset would only serve to introduce a dependence of country j's consumption on risks that could be avoided by holding the diversified mutual fund of world traded goods.

To determine portfolio shares in the three mutual funds, recall that equation (Eq. 9) was of the form

$$\mathrm{d}x_j = \alpha^X \mathrm{d}X + \alpha^Z \mathrm{d}Z + \alpha^z \mathrm{d}z_j \tag{10}$$

with $\alpha^X = x/X = 1$, $\alpha^Z = -\eta^Z X$, and $\alpha^z = \eta^z (x/z)$. Eq. (10) shows how optimal allocations – and optimal portfolio returns – must respond to the three elemental sources of uncertainty facing country j: $\mathrm{d}X$, $\mathrm{d}Z$, and $\mathrm{d}z_j$.

3.1. Asset payoffs

The traded good is the natural numeraire for the world economy. We define a ''market security'' to be a claim to a dividend (i.e., the endowment of a particular good) denominated in units of the traded good. By contrast, we define a ''basic security'' to be a claim to a dividend (i.e., an endowment of a particular good) denominated in own-goods units. Thus Eq. (10) expresses the dependence of the optimal allocation of x_j on the three basic securities relevant for residents of country j. Our task at this point is to express optimal allocations in terms of market securities.

To begin, consider a portfolio representing a claim to the world endowment of the traded good, which we denote by $\mathbf{T}$. Recalling that ξ_j denotes country j's endowment of the traded good, we have $\mathbf{T} = \Sigma_{j=1}^{J} \pi_j \xi_j = X$, so that

$$\mathrm{d}\mathbf{T} = \mathrm{d}X$$

For generality and for comparison with the market securities considered below, we write the dependence of the payout on the world traded good portfolio on the three underlying sources of uncertainty as follows:

$$\mathrm{d}T = \Lambda_T^X \,\mathrm{d}X + \Lambda_T^Z \,\mathrm{d}Z + \Lambda_T^z \,\mathrm{d}z$$

with $\Lambda_T^X = 1$, $\Lambda_T^Z = 0$, and $\Lambda_T^z = 0$. We turn next to the market security representing an equity claim on country j's nontraded good. This market security pays out $\mathrm{d}\mathbf{n}_j \equiv \mathrm{d}(p_j z_j)$. That is: the equity claim on the nontraded good will vary with changes in z_j, but will also vary with changes in p_j, the relative price of z in terms of the traded good. Substituting for dp_j using equation (Eq. (5)) we have:

$$\mathrm{d}\mathbf{n}_j \equiv \mathrm{d}(p_j z_j) = \mu\left(\frac{pz}{x}\right) \mathrm{d}X - \mu(pz)\eta^Z \,\mathrm{d}Z + p(\mu\eta^Z + (1-\mu))\,\mathrm{d}z_j \tag{11}$$

Thus the equity claim on the domestic nontraded good industry has a payoff structure of the form:

$$\mathrm{d}\mathbf{n}_j = \Lambda_n^X \, \mathrm{d}X + \Lambda_n^Z \, \mathrm{d}Z + \Lambda_n^z \, \mathrm{d}z_j$$

with $\Lambda_n^X = \mu(pz/x)$, $\Lambda_n^Z = -\mu(pz)\eta^Z$, and $\Lambda_n^z = p(\mu\eta^Z + (1-\mu))$.

Finally, the market security representing an equity claim on the world endowment of the nontraded good is denoted by N and has payout equal to:

$$\mathrm{d}\mathbf{N} = \sum_{j=1}^{J} w_j \, \mathrm{d}\mathbf{n}_j$$

where we require that the portfolio weights sum to one: $\Sigma_{j=1}^{J} \; w_j = 1$. In the symmetric case under consideration, $w_j = \pi_j$ so that the expression for $\mathrm{d}\mathbf{N}$ simplifies to:

$$\mathrm{d}\mathbf{N} = \mu(pz) \frac{\mathrm{d}X}{X} + (1-\mu)(pz) \frac{\mathrm{d}Z}{Z}$$

Once again, the payout to this market security is of the general form:

$$\mathrm{d}\mathbf{N} = \Lambda_N^X \, \mathrm{d}X + \Lambda_N^Z \, \mathrm{d}Z + \Lambda_N^z \, \mathrm{d}z$$

with $\Lambda_N^X = \mu(pz/x)$, $\Lambda_N^Z = p(1-\mu)$, and $\Lambda_N^z = 0$.[5]

Because of the assumed (approximate) linearity of the dependence of each of the market securities on the underlying sources of uncertainty, we can express the three sources of shocks – the three basic securities – as the following linear combinations of the market securities:

$$\mathrm{d}X = \mathrm{d}\mathbf{T} \tag{12}$$

$$\mathrm{d}Z = \left(\frac{1}{\Lambda_N^Z}\right) \mathrm{d}\mathbf{N} + \left(-\frac{\Lambda_N^X}{\Lambda_N^Z}\right) \mathrm{d}\mathbf{T} \tag{13}$$

$$\mathrm{d}z_j = \left(\frac{\Lambda_j^Z}{\Lambda_j^z}\frac{\Lambda_N^X}{\Lambda_N^Z} - \frac{\Lambda_n^X}{\Lambda_n^z}\right) \mathrm{d}\mathbf{T} + \left(-\frac{1}{\Lambda_N^Z}\frac{\Lambda_n^Z}{\Lambda_n^z}\right) \mathrm{d}\mathbf{N} + \left(\frac{1}{\Lambda_n^z}\right) \mathrm{d}\mathbf{n}_j \tag{14}$$

That is: one can generally structure portfolios of market securities that replicate basic securities, using the weights in Eqs. (12)–(14).

3.2. Supporting optimal consumption

In order to purchase his optimal allocation, an individual living in country j must have expenditure (purchasing power in units of the traded good) equal to

$$e_j \equiv x_j + p_j z_j.$$

[5] These coefficients reflect the implication of symmetric equilibrium that $X = x$ and $Z = z$.

The displacement in expenditure arising from displacements in world and national endowments is given by

$$\mathrm{d}e_j = (\alpha^X + \Lambda_n^X)\,\mathrm{d}X + (\alpha^Z + \Lambda_n^Z)\,\mathrm{d}Z + (\alpha^z + \Lambda_n^z)\,\mathrm{d}z_j \tag{15}$$

In Eq. (15), the α coefficients represent the sensitivity of optimal allocations of x to the three elemental sources of uncertainty while the Λ coefficients represent the sensitivity of pz to these factors. The form of Eq. (15) suggests that one way to generate the income necessary to purchase one's optimal allocation is to hold appropriate quantities of the three basic securities. Having determined how to construct the basic securities as linear combinations of the market securities, it is straightforward to characterize the quantities of each of the market securities that an individual must hold in order to be able to purchase his optimal consumption basket. Substituting for dX, dZ, and dz_j from Eqs. (12)–(14), we have:

$$\mathrm{d}e_j = \vartheta^T\,\mathrm{d}\mathbf{T} + \vartheta^N\,\mathrm{d}\mathbf{N} + \vartheta^n\,\mathrm{d}\mathbf{n}_j \tag{16}$$

with:

$$\vartheta^T = 1 \tag{17}$$

$$\vartheta^N = -(\vartheta^n - 1) \tag{18}$$

$$\vartheta^n = 1 + \left(\frac{x}{pz}\right)\frac{n^Z}{\mu\eta^Z + (1-\mu)} \tag{19}$$

Units are chosen so that holding a portfolio share of 1 corresponds to a country's share in the world portfolio for that particular asset.

3.3. *Can home bias be optimal?*

At this point we can evaluate whether it can be optimal to display ''home bias'' in one's portfolio. Looking first at traded-good equities, we find that one should always hold a diversified world portfolio of traded good equities. Put differently, an individual's portfolio holdings of domestic traded-good equities should equal his country's share in the world portfolio – this is reflected in the portfolio loading $\vartheta^T = 1$. Evidently, it is never optimal to exhibit home bias in a portfolio of traded-good equity.[6]

With respect to nontraded good equities, things may be different. Specifically, we find that investors may hold more or less than 100% of domestic nontraded good equities, and that there is an important corresponding role for holding other countries' nontraded-good equity. Eq. (19) indicates the extent to which an

[6]This result is not dependent on the symmetry assumption: see Baxter et al. (1995) for discussion of the general case.

individual wishes to hold more or less than 100% of the claim on own nontraded goods. Eq. (18) makes clear that the optimal share in the world portfolio of nontraded-good equity is closely related to the deviation from the 100% holding of domestic nontraded-good equity. In terms of our general model, Stockman and Dellas (1989) studied a parametric special case that implied $\eta^Z=0$. In this case, the coefficients above simplify to the following: $\vartheta^T=1$; $\vartheta^n=1$; $\vartheta^N=0$. These coefficients imply that each individual receives his country's share of the world supply of the traded good, regardless of the realization of the nontraded good endowment in his country. Thus the supporting portfolio consists of the individual's domestic share of the world traded goods portfolio ($\vartheta^T=1$) together with a claim on the entire nontraded good endowment of his country ($\vartheta^n=1$). In this economy there is no benefit to holding claims to the world nontraded good portfolio ($\vartheta^N=0$) since the prior two components support the optimum.

In general, however, we find that investors may hold more or less than 100% of the claims to the domestic nontraded-good equity. Correspondingly, there is an important role for holding other countries' nontraded-good equities, as indicated in Eq. (19). Further, Eq. (18) makes clear that the optimal share in the world portfolio of nontraded-good equity is closely related to the deviation from the 100% holding of domestic nontraded-good equity.

By contrast, Tesar (1993) and Pesenti and van Wincoop (1994) have suggested that home bias in traded-good equities may arise in a world with nontraded goods. These authors arrive at this conclusion primarily because they assume that nontraded-good equities are not tradable internationally.

Tesar's analysis proceeds as follows. First, she considers an initial situation in which an investor holds his country's share in the world portfolio of traded-good equities, and (of necessity, due to the assumed lack of international tradability of these claims) also holds 100% of the equity claims on domestic nontraded goods. From the analysis of the present paper, we know that this portfolio cannot support the optimal plan unless $\eta^Z=0$. Starting from this suboptimal position, Tesar then asks whether the domestic consumer would be marginally better off if he could exchange a very small fraction of his holdings of other countries' traded-good equities for additional holdings of his own traded-good equities. Depending on preference parameters and return covariances, this perturbation from the initial situation can make the individual better off. On this basis, Tesar argues that such perturbations rationalize home bias in portfolio choice. However, the difficulty with this argument is that such perturbations do not achieve the optimum. Instead, as we have shown in the present paper, supporting the optimal consumption allocations generally requires individuals to take positions in traded-good equities and nontraded-good equities in both countries. Because trade in nontraded-good equities is ruled out ex ante in the Tesar analysis, we can be sure that her portfolio allocations do not, in general, support the optimum. In summary, the fact that home bias can produce a marginal improvement from an arbitrary initial position with no trade in nontraded-good equities does not imply that home bias would characterize the optimal portfolio in a situation with free trade in all assets.

The contribution of Pesenti and van Wincoop's work is to begin to combine aspects of incomplete markets and optimal portfolio choice. However, because of the absence of an important class of markets (there is no international trade in nontraded-good equities), their paper also does not provide a description of optimal portfolio choice with frictionless international trade in securities.

3.4. Interpreting the portfolio allocations

How can we understand the economic forces that shape the portfolio choice decision? Let's look more closely at the demand for domestic nontraded-good equities. We determined that domestic holdings of nontraded-good equities should be:

$$\vartheta^n = 1 + \frac{\alpha^z}{\Lambda_n^z} = 1 + \left(\frac{x}{pz}\right)\frac{\eta^Z}{\mu\eta^Z + (1-\mu)}$$

where the second equality uses the facts that $\alpha^z = \eta^z(x/z)$, $\Lambda_n^z = p(\mu\eta^Z + (1-\mu))$. Recall that units have been chosen so that $\vartheta^n = 1$ corresponds to holding 100% of domestic nontraded-good equities.

Fig. 1 illustrates how holdings of domestic nontraded good equities depend on

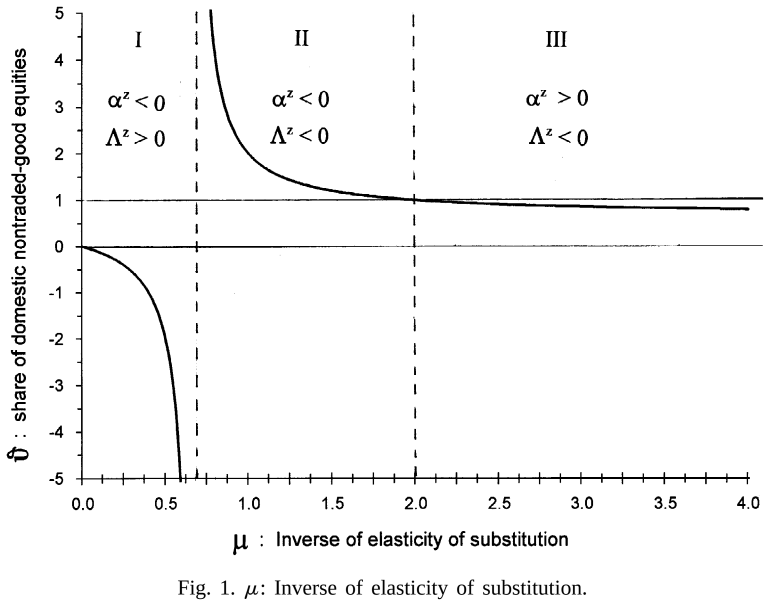

Fig. 1. μ: Inverse of elasticity of substitution.

the elasticity of substitution, $1/\mu$, between traded and nontraded goods. This figure is drawn under the assumptions that $\sigma=2$ and the consumption shares devoted to traded and nontraded goods are equal so that $x/pz=1$.

There are three distinct regions in Fig. 1. In region I, $\mu<\sigma$, so that $\alpha^z<0$. The low value of μ corresponds to a high elasticity of substitution between the traded and nontraded goods, so that the optimal allocation of x falls when a country's endowment of z is high. This is accomplished by creating a portfolio that has a low payoff when the endowment of z is high. Whether this involves a large or small position in the nontraded good portfolio depends on whether the nontraded good portfolio's payoff is high or low when the endowment is high. When $\Lambda^z>0$, as it is in region I, the nontraded good portfolio has a high payoff when the endowment of the nontraded good is high. Thus the portfolio weight on nontraded goods must be very small – in fact, negative – to deliver the required low payoff when the domestic nontraded good endowment is high.

In region II it is still the case that $\mu<\sigma$, so that $\alpha^z<0$. Thus the portfolio weight on nontraded-good equities will still be chosen to deliver a low payout when the nontraded good endowment is high. However, in region II we now have $\Lambda^z<0$, so that the payout to the nontraded good portfolio is low when the endowment is high. That is: the price of the nontraded good falls so much when the endowment of the nontraded good is high that the overall payout to the equity, pz, actually falls. Thus the desired sensitivity is achieved by holding large quantities – greater than 100% – of domestic nontraded good equities.

There is an asymptote that divides region I from region II. This asymptote occurs at the point at which Λ^z changes sign. At this particular point, the price and the quantity effect from an endowment shock in the nontraded-good industry exactly offset each other-the return to the domestic nontraded good portfolio does not depend on the endowment of the domestic nontraded good. In this case, the domestic nontraded good equity cannot be used to provide the required sensitivity of x to variations in the endowment of the z good. For this particular value of μ, the decentralized economy cannot support the optimal allocations. In particular, derivative securities would be needed to achieve complete markets.

Recent research by Ostry and Reinhart (1992) uses panel data from thirteen developing countries to estimate the elasticity of substitution between traded and nontraded goods. Their results suggest that a plausible range for μ is the interval (0.75,1.50) – the estimated value of the elasticity varied according to specific subgroup of countries studied and the instrument set employed. Their estimates of σ are in the neighbourhood of $\sigma=2$, which is consistent with results obtained by other researchers using macro data from developed countries. In terms of Fig. 1, the Ostry/Reinhart estimates suggest that $\mu<\sigma$, so that $\alpha^z<0$. Thus the regions of Fig. 1 most likely relevant for actual economies are regions I and II.

The dividing point between region II and region III is the case in which $\mu=\sigma$, implying that $\alpha^z=\partial x/\partial z=0$. In this case, each country holds all of the claims to its own nontraded goods equity because it is a perfect hedge for nontraded goods consumption: $\vartheta_j^n = 1$. In region III, $\mu > \sigma$, which means that traded and nontraded goods are poor substitutes. In this case $\alpha^z > 0$: optimal allocations of the traded good are high when then endowment of the domestic nontraded good is high. Thus the supporting portfolio must have a high payout when the endowment of the nontraded good is high. Since this region is characterized by $\Lambda^z < 0$ (payouts to the domestic nontraded good portfolio fall when the endowment rises), the desired sensitivity is obtained through a portfolio share in domestic nontraded-good equities that is positive but smaller than 100% ($0 < \vartheta^n < 1$). The results of Ostry and Reinhart (1992) suggest that this region is least likely to be empirically relevant.

4. Nontraded human capital

This section considers the role of risk associated with returns to nontraded human capital in the presence of nontraded goods. The analysis builds on that of Baxter and Jermann (1997) who show that the returns to human capital and physical capital are highly correlated within countries, but are weakly correlated across countries. To simplify the analysis in the present paper, we assume that the returns to human capital are perfectly correlated with the returns to domestic physical capital. The results would be little changed by assuming correlations in line with those estimated by Baxter and Jermann (1997). Since there is no traded asset whose payoff is explicitly contingent on the return to human capital (labour) in either sector, a claim to physical capital can be used to hedge the risk associated with nontraded human capital. We follow Black (1987) in identifying the traded claims on firms as the appropriate measure of claims to the payouts of physical capital. For concreteness, we call these traded claims the national ‘‘equity market.’’[7]

In the case of perfect correlation between labour and capital returns and a single, internationally traded consumption good, the hedge is constructed as follows. Let s_K denote the share of capital in aggregate output, and let $s_L = 1 - s_K$ denote labour's share (i.e., the share of human capital) in aggregate output. Since labour's share in the U.S. is approximately two–thirds, the national equity market represents about $s_K = 1/3$ of aggregate wealth, with nontraded human capital representing the remaining $s_L = 2/3$. Thus the hedge is established by selling short the domestic equity market in an amount equal to s_L of investor wealth, or $s_L/s_K = 2$ times the value of the national equity market. Having hedged the risk associated with nontraded human capital, the investor then constructs his optimal portfolio. A simple portfolio strategy is simply to diversify by holding the world portfolio of traded equities, where country j's weight in the (value-weighted)

[7]That is: we are thinking of the unlevered equity market, which would combine the stock market with the market for corporate debt.

world portfolio is given by Γ_j, with $\Sigma_{j=1}^{J} \Gamma_j = 1$. In this case, the net position held by the domestic investor in country j as a fraction of investor wealth is given by $\Gamma_j - s_L$. As a fraction of the national equity market the net position is $(\Gamma_j - s_L)/s_K$.

For the United States, which represented about $\Gamma_j = 1/2$ of world equity markets in 1991, the net position would be a short position equal to $(0.67-0.50)/0.33 = 0.51$ (51%) of the national equity market. For countries representing smaller fractions of the world equity market (smaller Γ_j), the short position in national equities would be even larger in absolute value. The U.K., for example, represented 15% of the world equity market in 1991, which would imply that a U.K. investor should hold a short position in U.K. equities of 158% of the equity market (again, assuming that labour's share is 2/3).

Intuitively, these results reflect the fact that nontraded human capital represents a very large fraction of aggregate wealth, so that large negative positions in national equities are needed to hedge the associated risk. So long as labour's share in a particular country exceeds the country's share in the world portfolio, the net position in national equities will be negative. Further, the short position will be larger (in absolute value) the smaller the country's share in the world portfolio.

How are these results modified when there are both traded and nontraded goods? For the traded good, these results go through directly. Section 3 showed that the optimal supporting portfolio involved holding exactly one's own country's share of the traded good. Let s_{LX} denote the share of labour income in the traded goods industry, assumed for simplicity to be identical across countries. In the presence of nontraded human capital, the investor receives a share s_{LX} of the payoff from the domestic traded good as a nontraded return to human capital. The remainder of the payoff from the domestic traded good, $s_{KX} = (1 - s_{LX})$, is the payoff to the traded claim on the domestic traded good. Since the investor simply wants to hold a claim to the world endowment of the traded good, he follows exactly the procedure described above. First, he hedges the nontraded labour income by establishing a short position in the domestic traded-good equity equal to $s_{LX}\xi_j / s_{KX}\xi_j = s_{LX}/s_{KX}$ (as a fraction of the market in domestic traded-good equity). He then would use the proceeds to purchase the world traded good portfolio, of which his country's share is Γ_j. Thus, as before, his net position in the domestic traded-good equity, as a fraction of that market, is given by $(-s_{LX} + \Gamma_j)/s_{KX}$. Assuming the labour's share in traded goods is approximately the same as it is in aggregate output, the analysis above suggests a short position in national traded-goods equities of about 51% for the United States, and about 158% for the U.K.

Now, consider how the presence of nontraded human capital alters the optimal holdings of the equity claim on domestic nontraded goods. In Section 3 we showed that the optimal portfolio contained a fraction $\vartheta^n = 1 + \kappa$ of the domestic nontraded good which, in the symmetric case is equal to

$$\vartheta^n = 1 + \kappa = 1 + \left(\frac{x}{pz}\right) \frac{\eta^Z}{\mu\eta^Z + (1-\mu)}$$

In the nontraded goods sector, labour receives the share s_{LZ} of sectoral output, and capital receives $s_{KZ}=(1-s_{LZ})$. If the individual holds 100% of the equity claim on the nontraded goods sector together with the nontraded labour income from this sector, his holdings of the nontraded good are $\vartheta_j^n=1$. To achieve the optimal holdings $\vartheta^n=1+\kappa$ (as a fraction of investor wealth), the investor must purchase additional units of the domestic nontraded good equity in the amount κ/s_{KZ}.

To take a specific example, consider the case in which $\mu<\sigma$ so that $\eta^Z<0$, and $\vartheta^n<1$, or $\kappa<0$ (this point is located in region I of Fig. 1). Suppose that, absent human capital considerations, $\kappa=-1.20$, so that the optimal holdings of the domestic nontraded good equity represented -20% of the market in that equity. When we introduce nontraded human capital, the optimal portfolio share in the domestic nontraded good equity drops to $\vartheta^n=1+\kappa/s_{KZ}=1+(-1.20/0.33)=0.40$.

Alternatively, suppose that $\vartheta^n=1.15$ absent human capital considerations (implying that this is a point in region II of Fig. 1, with $\kappa=0.15$). In this case, incorporating human capital increases the optimal share in the domestic nontraded good equity rises from $\vartheta^n=1.15$ to $\vartheta^n=1+\kappa/s_{KZ}=1.45$.

To summarize, there are two important effects of incorporating nontraded human capital. First, for holdings of the equity associated with the traded good, the Baxter-Jermann analysis goes through unchanged. So long as labour's share exceeds the nation's share in the world equity market, as seems plausible for every country in the world, the net position in the country's traded good equity will be negative. Further, this short position will be larger (in absolute value) the smaller the country's share in the world equity market. Second, the presence of nontraded human capital amplifies the deviation of the optimal portfolio holdings of the nontraded good equity from a benchmark in which the investor holds 100% of equities on the nontraded good. That is: if the investor holds less than 100% of domestic nontraded good equity absent human capital considerations, say $100\times(1-\kappa)\%$, he will hold approximately $100\times(1-(s_{LZ}/s_{KZ})\kappa)\%$ once nontraded human capital is incorporated into the analysis.

5. Conclusion

This paper has studied the role of nontraded goods in generating home bias in investor portfolios. Specifically, we studied the problem of optimal portfolio choice in a world economy in which each country has a traded and a nontraded good. As in prior studies, we restrict attention to the case in which the quantities of these goods arrive as endowments. In contrast to earlier studies, however, we adopt a two-stage strategy. First, as in standard general equilibrium theory, we explicitly characterize Pareto optimal allocations of the traded good across countries. Second, we search for portfolios of securities that can support the optimal allocations in a decentralized setting.

Our main finding is that nontraded goods are unlikely to rationalize home bias. That is: our model predicts that investors should hold a diversified world portfolio of traded-good equities, so that their holdings of their own country's traded-good equities will typically be very small. Holdings of domestic nontraded-good equities depends sensitively on the substitutability between traded and nontraded goods. When we consider the portfolio implications of incorporating nontraded human capital, we find that hedging the risk associated with the nontraded human capital likely involves an overall short position in domestic traded-goods equities. For nontraded goods, the incorporation of human capital amplifies the deviation from a benchmark in which an investor holds 100% of domestic nontraded-good equities.

In conclusion, our results suggest that home bias is not a rational response by investors to an environment with both traded and nontraded goods. Rather, these results suggest that there should be important gains to international diversification, even in the presence of nontraded consumption goods.

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