"Crises & Prices: Info. Aggregation, Multiplicity & Volatility"
Angeletos & Werning (AER, 2006)

- Extends Morris & Shin to include public info.

- Considers 2 main cases: Exogenous public info.
  Endogenous public info. (prices)

Main Results: 1.) Observation of public signals may restore multiple equilibria

2.) With exog. public signal, multiplicity more likely when public signal is precise relative to private signal.

3.) With endog. public signal multiplicity more likely. Public signal endogenously becomes more precise as precision of private signal increases.

Intuition: Public info. facilitate coordination, private info. impedes it. If public info. precision increases faster than private info. precision as noise decreases, then coordination becomes possible.

Case 1 : Exog. Public Signal

Assumptions :

1.) Measure 1 continuum of agents. Agents can choose 2 actions, "attack" ($a_i = 1$), "no attack" ($a_i = 0$)

2.) Govt. abandons peg if $A > \theta$

θ : fundamentals
A : mass of attackers

3.) $u(a_i, A, \theta) = a_i(1_{A>\theta} - c)$

c : transaction cost

Note,

a.) $u(1, A, \theta) - u(0, A, \theta)$ increases in A [actions are strategic complements]

b.) If θ is CK, then both $A=0$ and $A=1$ are Nash equil.

4.) Agent i receives private signal $x_i = \theta + \sigma_x \varepsilon_i$ $\quad \varepsilon_i \sim N(0,1)$

also receives public signal $z = \theta + \sigma_z \varepsilon$ $\quad \varepsilon \sim N(0,1)$

Equilibrium

Look for a symmetric perfect Bayesian equil. in threshold policies.

- Agent attacks iff $x < x^*(z)$ > Note, threshold now depends on realization of public signal

- Given this policy,

$$A(\theta, z) = \Pr[x \le x^*(z) \mid \theta] = \Phi(\sqrt{\alpha_x}(x^*(z) - \theta)) \qquad \alpha_x = \frac{1}{\sigma_x^2}$$

= signal precision

- Note, peg abandoned iff $\theta \le \theta^*(z)$ where $\theta^*(z)$ solves the consistency condition $A(\theta, z) = \theta$.

- Using the above expression for $A(\theta, z)$ we get

(1) $$\boxed{x^* = \theta^* + \frac{1}{\sqrt{\alpha_x}} \Phi^{-1}(\theta^*)}$$ > Govt's Indifference Condition

- Expected Profit from attacking is,

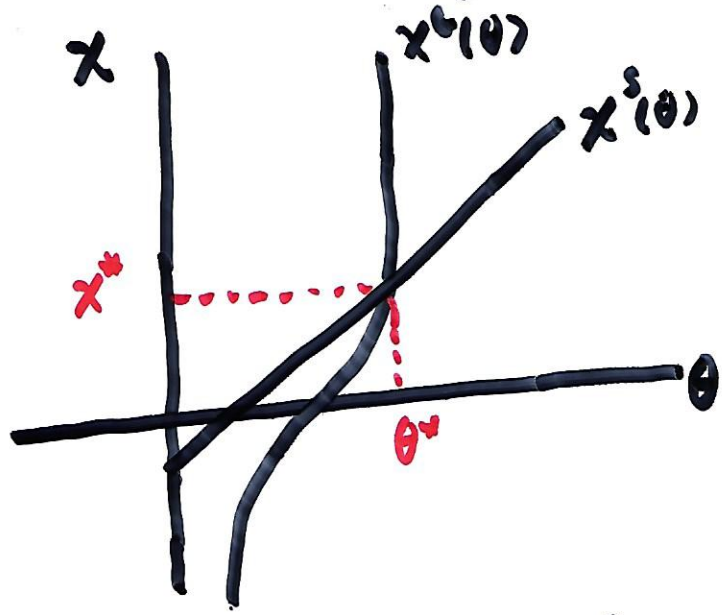
$$\Pr[\theta \le \theta^*(z) \mid x, z] - c$$

- Note, posterior for θ is $N\left(\frac{\alpha_x}{\alpha} x + \frac{\alpha_z}{\alpha} z, \frac{1}{\alpha}\right)$ $\quad \alpha = \alpha_x + \alpha_z$

$$\Rightarrow \Phi\left(\sqrt{\alpha}\left(\theta^*(z) - \frac{\alpha_x}{\alpha} x^*(z) - \frac{\alpha_z}{\alpha} z\right)\right) = c$$

$$\Rightarrow$$ (2) $$\boxed{x^* = \frac{\alpha}{\alpha_x} \theta^* - \frac{\alpha_z}{\alpha_x} z - \frac{\sqrt{\alpha}}{\alpha_x} \Phi^{-1}(c)}$$ > Speculators' Indifference condition

- Equations (1) & (2) jointly determine (x^*, θ^*)

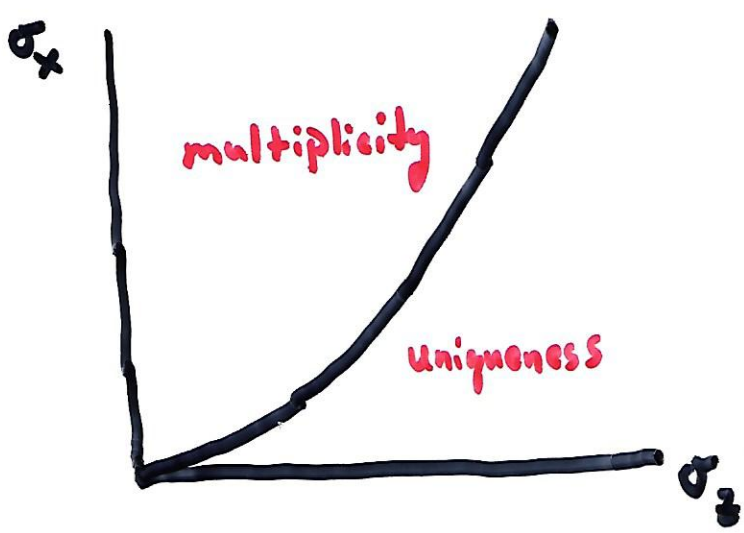


- Equil. will be unique if x^S is alway flatter than x^G

$$\frac{\partial x^S}{\partial \theta} = \frac{\alpha}{\alpha_x} \qquad \frac{\partial x^G}{\partial \theta} = 1 + \frac{1}{\sqrt{\alpha_x}} \frac{1}{\partial \Phi / \partial x}$$

$$\frac{\alpha}{\alpha_x} < 1 + \frac{1}{\sqrt{\alpha_x}} \min\left(\frac{1}{\partial \Phi / \partial x}\right) = 1 + \frac{1}{\sqrt{\alpha_x}} \frac{1}{1/\sqrt{2\pi}}$$

$$\Rightarrow \boxed{\sigma_x < \sigma_z^2 \sqrt{2\pi}} \Longrightarrow \text{uniqueness}$$



Case 2: Endogenous Public Signal

- Agents interact in 2 stages
- In the 1st stage, they trade a risky asset with dividend $d = \theta$ at price p.
- Preferences $\max_{k_i} V(w_i) = -e^{-\gamma w_i}$

  w_0 = initial wealth

  s.t. $w_i = w_0 + (\theta - p)k_i$
- Supply of asset is random + unobserved,

  $K^s = \sigma_\varepsilon \cdot \varepsilon \qquad \varepsilon \sim N(0,1)$
- $k(x,p) = \dfrac{E[\theta \mid x,p] - p}{\gamma \, var[\theta \mid x,p]}$ $\Big\}$ $E[\theta \mid x,p] = \frac{\alpha_x}{\alpha} x + \frac{\alpha_p}{\alpha} p$, $\alpha = \alpha_x + \alpha_p$

$$\Rightarrow k(x,p) = \frac{\alpha_x}{\gamma}(x - p)$$

$$\Rightarrow K^d(x,p) = \frac{\alpha_x}{\gamma}(\theta - p) = K^s = \sigma_\varepsilon \cdot \varepsilon \quad \Big\} \text{ market clearing}$$

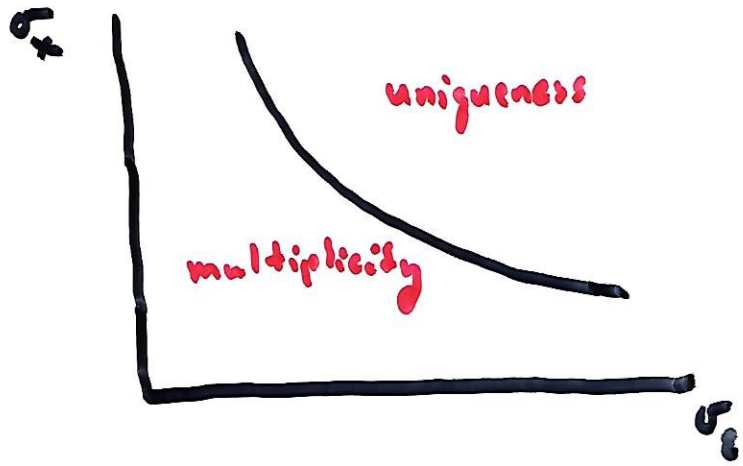
$$\Rightarrow \boxed{p = \theta - \underbrace{\gamma \sigma_\varepsilon \sigma_x^2}_{\text{equil. variance of public signal}} \cdot \varepsilon}$$

- The 2nd stage is the same as before,
  - agents choose whether to attack conditional on (p, x)
  - govt " " " abandon " " (A, d)

- Before, $\exists$ multiplicity (for some z) if $\sigma_x > \sigma_z^2 \sqrt{2\pi}$

- Now, $\sigma_z^2 = \gamma^2 \sigma_\varepsilon^2 \sigma_x^4$. Subbing in,

$$\boxed{\sigma_z^2 \sigma_x^3 < \frac{1}{\gamma^2 \sqrt{2\pi}}} \Rightarrow \text{multiple equil.}$$



- Before, uniqueness occurred as $\sigma_x \to 0$ because σ_z was <u>fixed</u>. Now, as $\sigma_x \to 0$, $\sigma_z \to 0$ <u>faster</u>

- Better private info $\Rightarrow$ better public info via trade in risky asset

- **2 extensions**

1.) Endogenous dividends / fundamentals

$$d(A) = -\Phi^{-1}(A)$$

$A\uparrow \Rightarrow$ fundamentals get worse

Same results as before except now prices can be nonunique

$$P\uparrow \Rightarrow \text{Less likely to attack} \Rightarrow \text{Higher dividends} \Rightarrow P\uparrow$$

2.) Noisy observation of others' actions

$$y = S(A, \varepsilon) = \Phi^{-1}(A) + \sigma_\varepsilon \cdot \varepsilon$$

Equil. Conds.

$$a(x,y) = \arg\max_{a \in \{0,1\}} E[u(a, A(\theta, y), \theta) \mid x, y] \quad \Big\} \text{ optimality}$$

$$A(\theta, y) = E[a(x,y) \mid \theta, y] \quad \Big\} \text{ Aggregation}$$

$$y = S(A(\theta, y), \varepsilon) \quad \Big\} \text{ Consistency RE Fixed Pt. Condition}$$

$\Rightarrow$ Multiplicity if $\sigma_\varepsilon^2 \cdot \sigma_x < \frac{1}{\sqrt{2\pi}}$