Labor Supply Examples

- We have 2 equations

$$\frac{u_\ell}{u_c} = w/p$$

$$c + w/p\,\ell = w/p$$

in 2 unknowns (c, ℓ).

① $\boxed{u(c,\ell) = \sqrt{c} + \alpha\sqrt{\ell}}$

$$\Rightarrow u_\ell = \tfrac{1}{2}\frac{\alpha}{\sqrt{\ell}} \qquad u_c = \tfrac{1}{2}\frac{1}{\sqrt{c}}$$

$$\Rightarrow \frac{u_\ell}{u_c} = \alpha\sqrt{c/\ell} = w/p \Rightarrow c/\ell = \frac{1}{\alpha^2}(w/p)^2$$

$$\Rightarrow c = \frac{1}{\alpha^2}(w/p)^2\,\ell$$

Sub into budget constraint

$$\frac{1}{\alpha^2}(w/p)^2\ell + w/p\,\ell = w/p \Rightarrow \boxed{\ell = \frac{1}{1 + \frac{1}{\alpha^2}w/p}}$$

$$\Rightarrow \boxed{N^s = 1 - \ell = \frac{w/p}{\alpha^2 + w/p}}$$

Note: $\frac{dN^s}{d(w/p)} > 0 \qquad \frac{dN^s}{d\alpha} < 0$

(2.) $U(c,\ell) = \ln(c) + \alpha \ln(\ell)$

$\Rightarrow U_\ell = \alpha/\ell \qquad U_c = 1/c$

$\Rightarrow \frac{U_\ell}{U_c} = \alpha \frac{c}{\ell} = w/p \Rightarrow \alpha c = w/p\, \ell$

Sub into budget constraint

$c + \alpha c = w/p \Rightarrow \boxed{c = \frac{1}{1+\alpha} w/p}$

$\Rightarrow \boxed{\ell = \frac{\alpha}{1+\alpha}} \Rightarrow N^s = \frac{1}{1+\alpha}$

Note! $\frac{dN^s}{d(w/p)} = 0$ Interpretation ?