A Simple Model of the Current Account

- Last time we saw that a nation's current acct. balance reflects a discrepancy between its saving and investment:

$$\boxed{CA = S - I}$$

To understand current accts. + capital flows, we therefore need a dynamic model.

- It is useful to think of a small open-economy operating in the global capital market exactly like an individual household operating in a (closed) domestic capital market. As with households, the key intuition is consumption-smoothing, and is formalized by Friedman's "Permanent Income Hypothe

- Our basic strategy is to begin slowly, with the simplest possible model, and then add real-world complications gradually, one at a time.

Assumptions

1.) Only 2 periods

2.) Small Open Economy (exogenous interest rate)

3.) Endowment Economy (No production)

4.) 1 good (only intertemporal gains from trade).

5.) No Uncertainty / 1 asset

6.) Representative Agent (Domestic markets are complete)

Budget Constraints

$$C_1 + B_1^* - B_0^* = r_0 B_0^* + Q_1 \quad \} \text{ period 1}$$

$$C_2 + B_2^* - B_1^* = r_1 B_1^* + Q_2 \quad \} \text{ period 2}$$

Note, the country has 2 sources of income: 1.) Q: output and 2.) rB: investment income. Likewise, it has 2 uses for income: 1.) C: consumption, and 2.) $B_t - B_{t-1}$: saving/asset accumulation.

"Transversality Condition"

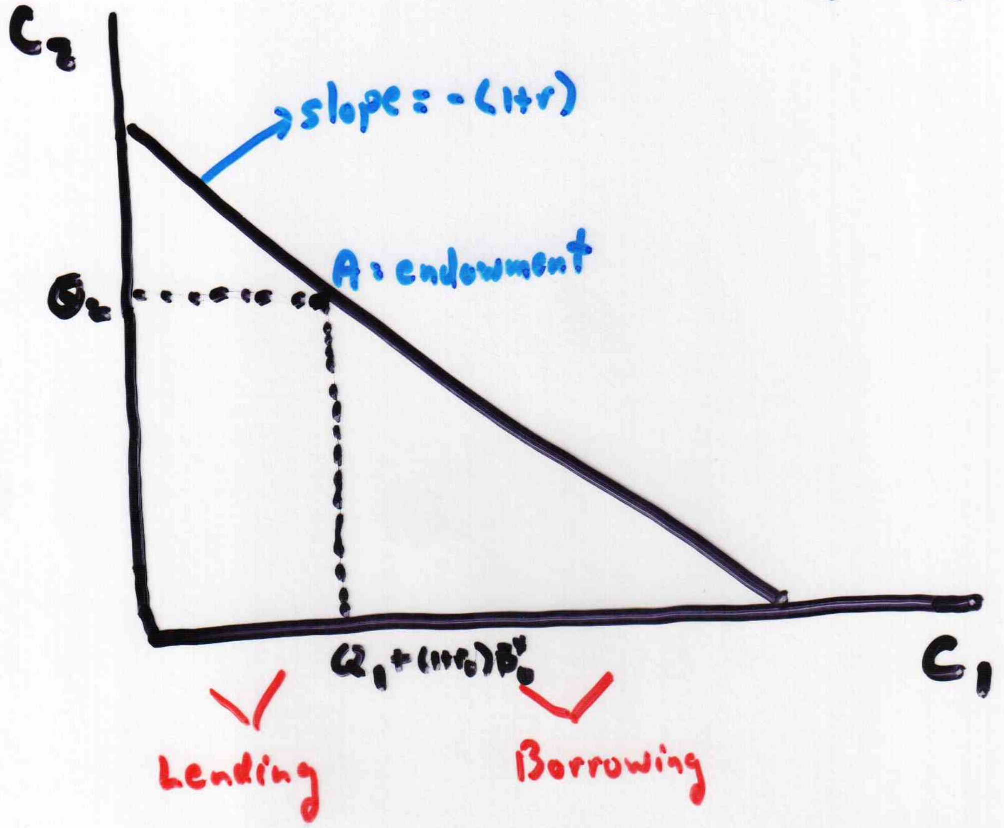
Since the world ends after period 2, a country would never choose $B_2^* > 0$. Doing so would be throwing money away! At the same time, it is not possible for $B_2^* < 0$ (why?). Hence, we have the transversality condition, $B_2^* = 0$.

- Imposing the transversality condition, and then substituting period 2's budget constraint into period 1's budget constraint, we get the following intertemporal budget constraint:

$$\boxed{C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}}$$

We can graph this as follows,

$$c_2 = Q_2 - (1+r_1)[(1+r_0)B_0^* + Q_1 - c_1$$



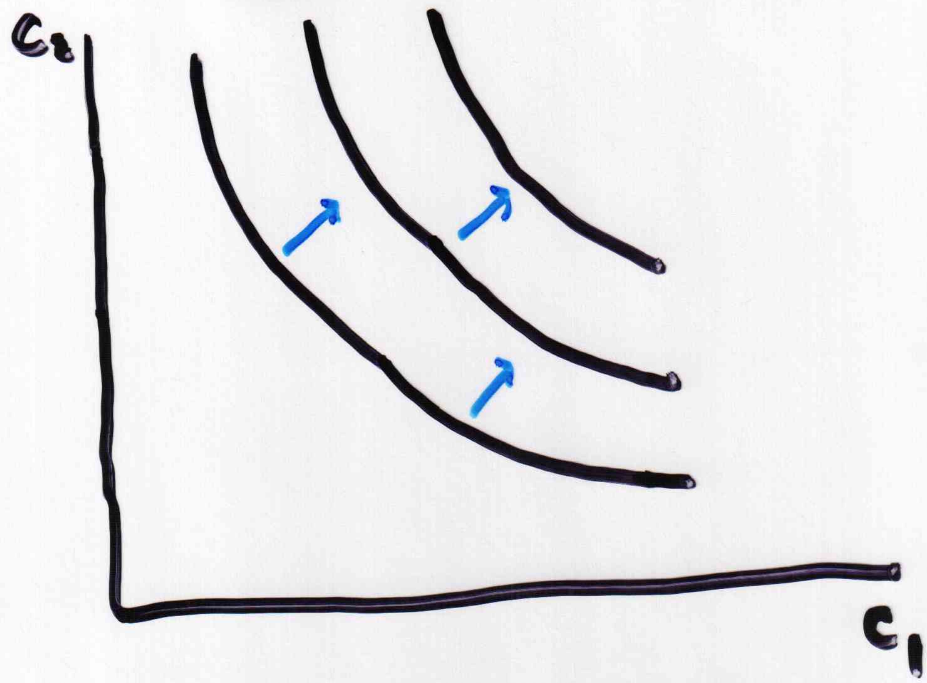
To determine which point the country chooses, we need to consider its preferences

Preferences

Preferences can be described by a "utility function"

$$U(c_1, c_2)$$

The essential aspects of preferences are contained in its "Indifference Curves".



Why are the indifference curves downward sloping and "convex"?

Along an indifference curve we have

$$du = \frac{\partial u}{\partial c_1} dc_1 + \frac{\partial u}{\partial c_2} dc_2$$

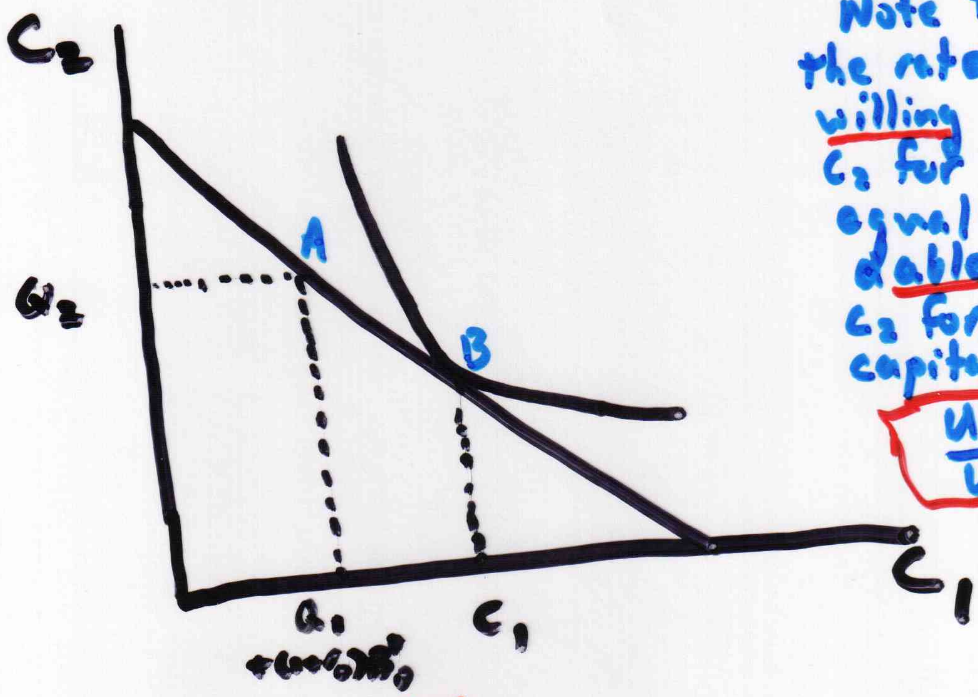
$$= u_1 \, dc_1 + u_2 \, dc_2 = 0$$

$$\Rightarrow \boxed{\frac{dc_2}{dc_1} = -\frac{u_1}{u_2}}$$

The slope of an Indifference Curve, $-\frac{u_1}{u_2}$, is called the Marginal Rate of Substitution.

It tells you how willing the consumer is to substitute C_2 for C_1. Convexity of the indifference curves reflects a "diminishing marginal rate of substitution".

The country's goal is to reach the highest Indifference Curve, while staying within its budget constraint. It is geometrically obvious that this occurs at a point of tangency between the Indifference Curve and the budget constraint [at pt. B below]:



Current Acct. Deficit

Note that in equilibrium the rate the country is willing to substitute C_2 for C_1 is exactly equal to the rate it is able to substitute C_2 for C_1 on the world capital mkt. That is,

$$\boxed{\frac{U_1}{U_2} = 1+r}$$

Note that this country runs a Current Acct. deficit because it consumes, C_1, more than its 1st period income, $Q_1 + (1+r_0)B_0^*$.

That is, it chooses to borrow.

An Algebraic Example

Suppose we have

$$U(c_1, c_2) = \ln c_1 + \ln c_2$$

Notice that in this case,

$$U_1 = \frac{1}{c_1} \qquad U_2 = \frac{1}{c_2}$$

Thus, the FOC/optimality condition is:

$$\frac{c_2}{c_1} = 1 + r_1 \implies c_2 = (1+r_1)c_1$$

Substituting into the intertemporal budget constrai

$$c_1 + c_1 = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}$$

$$= \bar{Y} \quad \text{(present discounted value of wealth)}$$

$$\implies c_1 = \tfrac{1}{2}\bar{Y}$$

Therefore,

$$TB_1 = Q_1 - c_1 = \frac{1}{2}\left[Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r_1}\right]$$

$$CA_1 = r_0 B_0^* + TB_1 = r_0 B_0^* + \frac{1}{2}\left[Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r_1}\right]$$

Current Account Sustainability

- Popular press commentators often assert that a given current acct. imbalance is "unsustainable". What exactly does this mean? One interpretation is that countries cannot run current acct. deficits forever. Eventually, a current acct. deficit must be matched by future surpluses. Is this true?

- Perhaps surprisingly, our very simple model allows us to answer these questions.

Finite Horizons

First consider the case of finite horizons, for simplicity, just 2 periods. Also assume for simplicity that the world interest rate, r, is constant

By definition, we know

(1) $CA_1 = rB_0^* + TB_1$

(2) $CA_1 = B_1^* - B_0^*$

Eliminating CA_1 from (1) and (2) gives

(3) $B_1^* = (1+r)B_0^* + TB_1$

For period 2 this gives

(4) $B_2^* = (1+r)B_1^* + TB_2$

Eliminating B_1^* from (3) and (4), and imposing the transversality condition, $B_2^* = 0$

(5) $\boxed{(1+r)B_0^* = -TB_1 - \frac{TB_2}{1+r}}$

Already we have an interesting result:

Unless a country has a positive initial net foreign asset position ($B_0^* > 0$), it is not possible for a country to run perpetual trade deficits. A period of trade deficits must be offset by future trade surpluses. On the other hand, if a country does have a positive initial net foreign asset position, it can run a perpetual TB deficit.

What about the Current Acct. ?

Again by definition,

$$CA_2 = B_2^* - B_1^*$$

Using (2) to eliminate B_1^* and imposing $B_2^* = 0$,

$$\boxed{B_0^* = -CA_1 - CA_2}$$

Again we have the same result,

A country cannot run perpetual Current Acct. deficits unless it has a positive initial net foreign asset position.

Infinite Horizons

- The previous results seem to support the popular press dictum that countries cannot run perpetual CA deficits.
- Interestingly, we shall now see that if the economy lasts forever, then it *is* possible to run perpetual CA deficits, even when the economy begins with zero net foreign assets!

Rewriting (3) gives us,

$$(6)\quad B_0^* = \frac{B_1^*}{1+r} - \frac{TB_1}{1+r}$$

Shifting forward one period

$$B_1^* = \frac{B_2^*}{1+r} - \frac{TB_2}{1+r}$$

Sub into (6)

$$B_0^* = \frac{B_2^*}{(1+r)^2} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2}$$

Iterating forward for T periods,

$$B_0^* = \frac{B_T^*}{(1+r)^T} - \frac{TB_1}{(1+r)} - \frac{TB_2}{(1+r)^2} - \cdots - \frac{TB_T}{(1+r)^T}$$

Now the Transversality Condition is more subtle. It must prevent "Ponzi Games", where current debts are paid by issuing new debt. It precludes,

$$\lim_{T\to\infty} \frac{B_T^*}{(1+r)^T} < 0$$

At the same time, it would not be optimal to have $\lim_{T\to\infty} \frac{B_T^*}{(1+r)^T} > 0$. Hence, the Transversality Condition becomes,

$$\boxed{\lim_{T\to\infty} \frac{B_T^*}{(1+r)^T} = 0}$$

Note: This does not require a country to "pay off" it debt. It just prevents debt from growing too fast.

Letting $T \to \infty$ and imposing the TVC,

$$B_0^* = -\sum_{j=1}^{\infty} \frac{TB_j}{(1+r)^j}$$

So again, even with infinite horizons, it is not possible to run perpetual TB deficits without having a positive initial net foreign asset position.

What about the Current Acct.?

Suppose the economy begins with $B_0^* < 0$. The no-Ponzi game condition prevents the country from rolling over its entire interest payment each period, so suppose the country pays a fraction, α, of its interest obligations each period, which requires it to run a TB surplus

$$TB_t = -\alpha r B_{t-1}^*$$

With this debt-servicing plan, the CA is as follows

$$CA_t = rB^*_{t-1} + TB_t$$
$$= r(1-\alpha)B^*_{t-1}$$

And foreign debt evolves as:

$$B^*_t = (1+r)B^*_{t-1} + TB_t$$
$$= (1+r-\alpha r)B^*_{t-1}$$

$$\Rightarrow B^*_t = (1+r-\alpha r)^t B^*_0$$

Notice that since $B^*_0 < 0$:

1.) The country runs perpetual CA deficits

2.) The country never "pays off" its foreign debt. $B^*_t < 0 \quad \forall t$

However, the key question is whether the TVC holds! Since $B_t^* = (1+r-\alpha r)^t B_0^*$ we have:

$$\frac{B_t^*}{(1+r)^t} = \left[\frac{1+r(1-\alpha)}{1+r}\right]^t B_0^*$$

Notice that since $1+r > 1+r(1-\alpha)$ we have

$$\lim_{t\to\infty} \frac{B_t^*}{(1+r)^t} \Rightarrow 0 \Rightarrow \text{TVC holds!}$$

The final issue is whether it is possible for the country to run the required TB surpluses.

By definition,

$$TB_t = B_t^* - (1+r)B_{t-1}^*$$

$$= -\alpha r\left[1+r(1-\alpha)\right]^{t-1} B_0^*$$

This implies the TB grows at rate $r(1-\alpha)$. Obviously, the trade surplus cannot exceed GDP! Therefore, for this to be feasible, it must be the case that

$$\boxed{g > r(1-\alpha)}$$

where g is the economy's growth rate.

This can be expressed as a minimum payment condition:

$$\boxed{\alpha > \frac{r-g}{r}}$$

Notice that the faster the economy grows the smaller is the required debt service.

Although it might appear as if no debt service at all is required if $g > r$, it turns out that $g > r$ is not consistent with our original small country assumption. (Countries that grow faster than the interest rate for long periods eventually become large! Remember, r is an equilibrium price, and it cannot be lower than all countries growth rates.)