Topics for Today

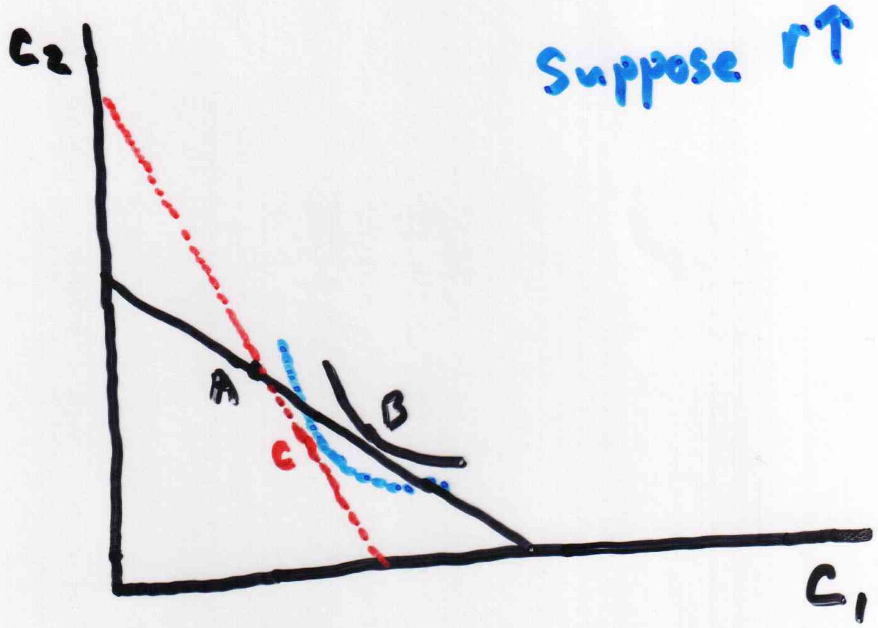
1.) World Interest Rate Shocks

2.) Investment, Production + the Current Account

- Production Functions + Profit Maximization
- Equilibrium in Closed Economies

World Interest Rate Shocks

- We have one loose end from last time - How does the CA respond to world interest rate shocks in an endowment economy?
- We can visualize this as follows:



A = endowment point

B = initial equil. (CA deficit)

C = new equil. (smaller CA deficit)

- We can decompose the shift from B to C into 2 parts:

1.) **Substitution Effect**: A (hypothetical) movement along the original Indifference Curve to a point of tangency with the new budget constraint. Note, this necessarily leads to $C_1 \downarrow$.

$r\uparrow \Rightarrow$ relative price of $C_1 \uparrow$
$\Rightarrow$ people substitute toward future consumption

Intuitively, when $r\uparrow$ the rate of return to saving rises, so people save more.

2.) **Income Effect**: Since the country was assumed to be borrowing initially ($CA<0$), an interest rate increase represents a negative income effect $\Rightarrow C_1\downarrow$ and $C_2\downarrow$. In this case, the income effect reinforces the substitution effect, and so $C_1\downarrow$ unambiguously. Since Q_1 is fixed, we know the CA deficit shrinks.

- What if the country had initially had a CA surplus?

- What if $r\downarrow$ instead?

Investment, Production + the Current Account

- Remember, $CA = S - I$. In an endowment economy, $I = 0$. Hence, so far we've only discussed half the story! We now take a major step toward realism by assuming that output must be produced. This gives an economy a little more flexibility, since it can reallocate output over time through investment.

- We make the following 3 assumptions:

1.) Capital is the only input to production. Labor inputs are ignored. Capital + output are the same good (one-sector production technology).

2.) Now what is given is the initial stock of capital.

3.) The output + factor markets are competitive. Profit-maximizing firms produce output. Households own the firms.

- Let K_1 : capital stock at beginning of period 1 (given)

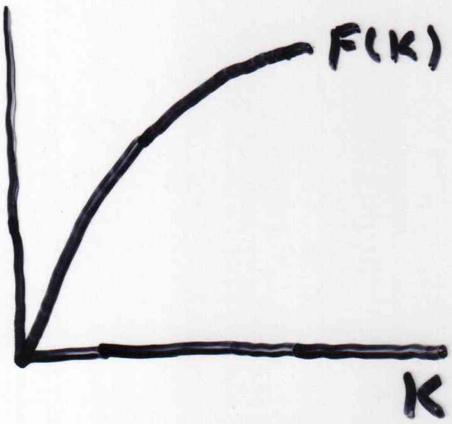
  K_2 = capital stock at beginning of period 2

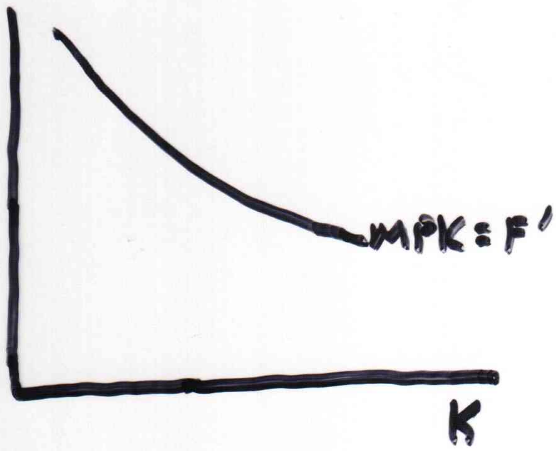
- Now output is an increasing function of capital

  $Q_1 = F(K_1)$ > since K_1 is fixed, so is Q_1

  $Q_2 = F(K_2)$

- We assume $F(\cdot)$ is increasing and concave, so that $MPK' < 0$ (i.e., $F' > 0$, $F'' < 0$).





- Capital depreciates at rate δ, so that

  $K_2 = (1-\delta) K_1 + I_1$

  where I_1 = period 1 gross investment

  (net investment = $K_2 - K_1$)

Firms

- Firms sell output at a price of 1 (output is the "numeraire"). Their costs of production consist of the rental rate on capital (sometimes called the "user cost of capital"). [Even if firms own their own capital, the rental rate represents the opportunity cost of capital].

- The rental rate consists of 2 parts:

  1.) The interest rate, r

  2.) The depreciation rate, δ

- Therefore, period 1 and 2 profits are:

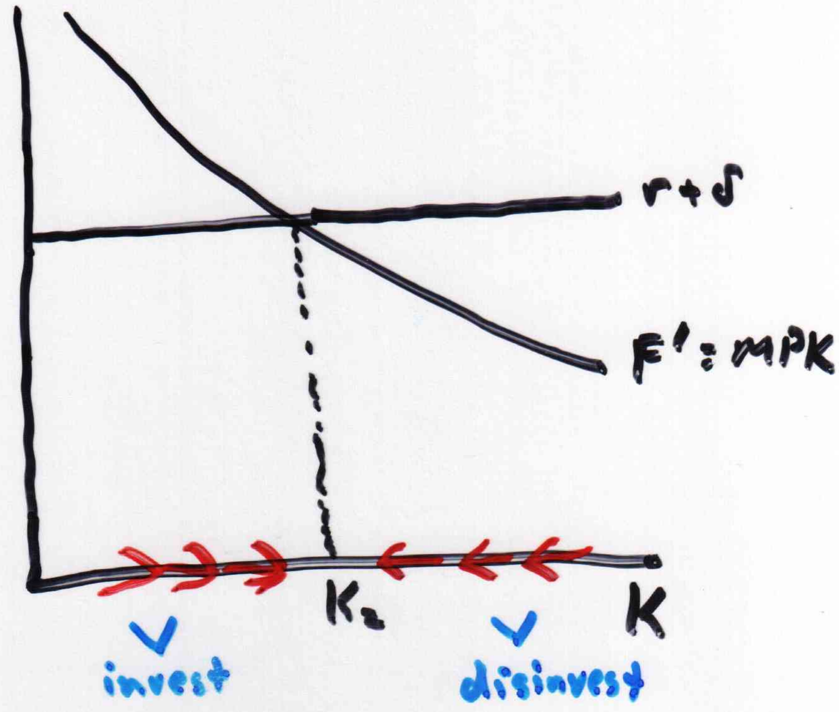
$$\pi_1 = F(K_1) - (r_0 + \delta) K_1 \qquad \text{(given)}$$

$$\pi_2 = F(K_2) - (r_1 + \delta) K_2$$

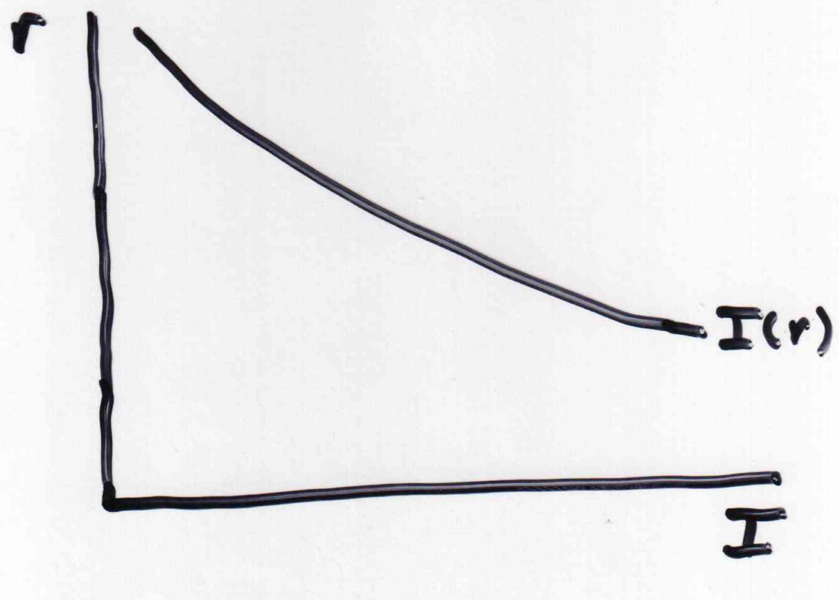
- Clearly, profits are maximized when

$$\boxed{F'(K_2) = r_1 + \delta}$$

- We can visualize this as follows:



- From the above graph, it is clear that $r \uparrow \implies K_2 \downarrow$. Since $I_1 = K_2 - (1-\delta)K_1$, we then know there is an inverse relationship between r and I



Households

- Households have 2 sources of income:

1.) Firm profits (dividends)

2.) Rental payments / asset income

- Households have 2 uses for income:

1.) Consumption

2.) Saving / Asset Accumulation

- Letting W_i = wealth at end of period i

$$\left.\begin{aligned} c_1 + (W_1 - W_0) &= r_0 W_0 + \pi_1 \\ c_2 + (W_2 - W_1) &= r_1 W_1 + \pi_2 \end{aligned}\right\} \text{budget constraints}$$

- Now the TVC becomes $W_2 = 0$. Therefore, the period 2 budget constraint becomes:

$$c_2 = (1+r_1)W_1 + \pi_2$$

- Substituting this into the period 1 budget constraint, we get the following intertemporal budget constraint

$$\boxed{c_1 + \frac{c_2}{1+r_1} = (1+r_0)W_0 + \pi_1 + \frac{\pi_2}{1+r_1}}$$

Equilibrium in a Closed Economy

- In a closed economy, the only form of wealth is the domestic capital stock:

$$W_0 = K_1$$
$$W_1 = K_2$$

- Substituting for Π_1 + Π_2 in the budget constraints, we recover the National Income Accounting identities:

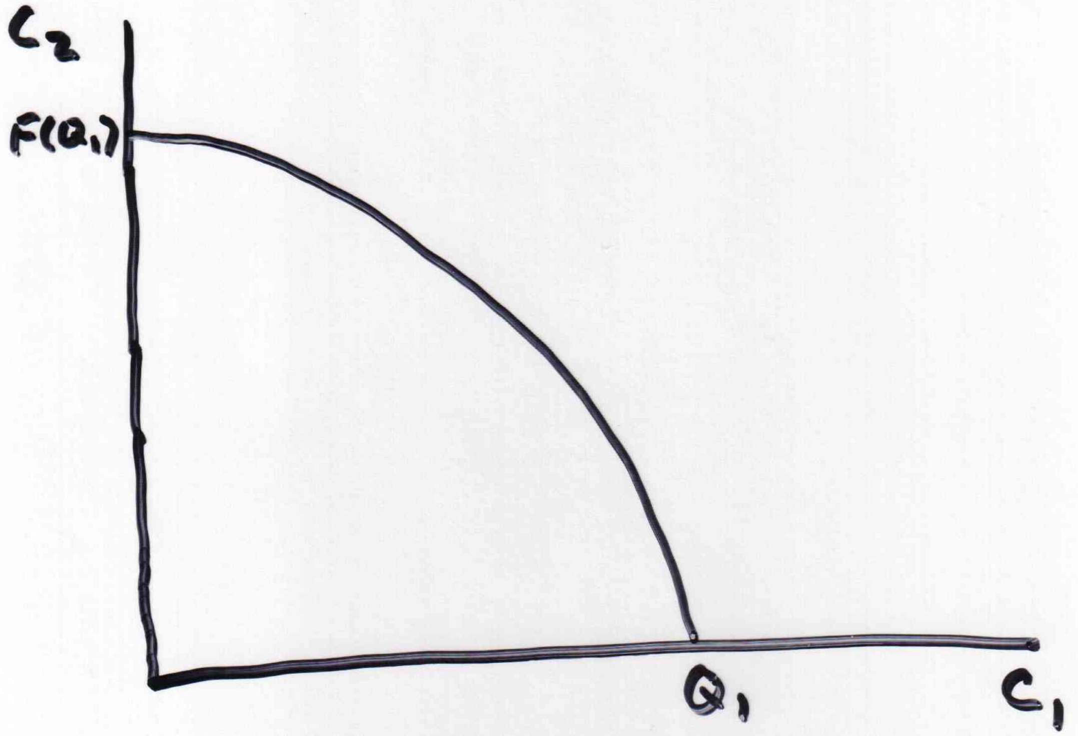
$$Q_1 = C_1 + K_2 - (1-\delta)K_1 = C_1 + I_1$$
$$Q_2 = C_2 - (1-\delta)K_2 = C_2 + I_2$$

- Since $Q_2 = F(K_2)$ and Q_1 is given, we can combine these two to get the economy's "Production Possibility Frontier" (PPF)!

$$\boxed{C_2 = F(Q_1 + (1-\delta)K_1 - C_1) + (1-\delta)[Q_1 + (1-\delta)K_1 - C_1]}$$

- When $\delta = 1$ this simply becomes

$$C_2 = F(Q_1 - C_1)$$



- When $\delta = 1$, the slope of the PPF is just $-F'(Q_1 - C_1)$

  When $\delta < 1$, the slope of the PPF is $-F'(\cdot) - (1-\delta)$

- In a closed economy, households must stay within the PPF

- In equilibrium, the slope of the Indifference Curve equals the slope of the PPF.

slope of IC $= -\dfrac{u_1}{u_2}$

slope of PPF $= -F'(\cdot) - (1-\delta)$

- Since $F'(\cdot) = r + \delta$, we have

$$\boxed{\frac{u_1}{u_2} = 1 + r}$$

- In a closed economy r is endogenous