Topic for Today

1.) The Diamond-Dybvig Model

- Recent events have shown how important the banking sector is to the economy, and what damage can occur when banks get into trouble.

- This raises some obvious questions:

1.) What exactly do banks do?

2.) What economic purpose do they serve?

3.) How would the economy function without them?

4.) Why are there periodic 'banking crises'?

5.) Can (and should) govts. do anything to prevent them?

- Today we develop the leading economic model of banking, called the "Diamond-Dybvig Model" (DD model). It attempts to answer the above questions.

Main Ingredients

- The DD Model features 2 critical assumptions:

1.) Households demand/value liquidity. They have random spending needs, and so they value immediate access to their funds.

2.) Many investment projects are illiquid. They take a long time to complete, and are worth less if terminated early.

- Since households are ultimately the providers of loanable funds in the economy, and firms are the primary demanders of loanable funds, there is a liquidity mismatch between the two sides of the market.

- Banks are institutions that absorb this mismatch. They create liquidity by issuing liquid deposits to households, while making illiquid loans to firms.

- Banks add value by exploiting the law of large numbers. To the extent that the random demand for liquidity is diversifiable, which will be the case if not everyone wants their money at the same time, then banks can pool this risk and offer liquidity insurance to households. In this sense, banks are just like insurance companies.

- Since banks invest in illiquid assets, which must be sold at a loss if liquidated early, banks will become insolvent if more than the expected number of depositors shows up to withdraw their money. Since everyone knows this, even those who don't need their money may find it in their interest to withdraw it if they think that everyone else is going to withraw their money. That is, banks are exposed to bank runs.

Assumptions

1.) The economy lasts for only three periods: 0, 1, and 2

2.) There is a large number, N, of consumers.

3.) Each consumer is endowed with one unit of a good, which can be invested in a production process.

4.) Investment projects are illiquid in the following sense: A unit invested in period 0 will yield $1+r$ units in period 2. However, if the project is interrupted in period 1, then only 1 unit is produced in period 1, and nothing after that.

5.) Consumers have random expenditure requirements. With probability t, a consumer will need to consume early, in period 1. If so, he is called a "type 1" consumer. With probability $(1-t)$ a consumer will be patient, and can wait until period 2 to consume. If so, he is called a "type 2" consumer.

6.) Consumers do not learn what type they are until the beginning of period 1. Each consumer's type is *private information*. There is no way it can be *verified* by an outside observer.

Autarky

Each consumer wants to maximize his *expected* utility:

$$\max_{c_1, c_2} \; t\,U(c_1) + (1-t)\,U(c_2)$$

where c_1 denotes consumption in period 1, and c_2 denotes consumption in period 2.

If left to his own devices, each consumer's decision is simple - Invest in period 0, and then terminate the project in period 1 if you discover you are type 1. Otherwise, wait until period 2 to consume. (Note: Although type 2's *could* terminate the project early if they wanted to, this would not be optimal).

A consumer's expected utility is therefore,

$$t\,U(1) + (1-t)\,U(1+r)$$

Banks

- Now the question is - Can 'the market' improve on this outcome? Could competitive, profit maximizing, institutions be organized that offer contracts to households that make them better off, in the sense of increasing their expected utility?

- If banks are competitive, then effectively they offer contracts that maximize consumers' expected utility subject to a break even or zero (expected) profit constraint.

- These contracts are of the following form:

  1.) If you withdraw in period 1, then you get d_1 units of the good.

  2.) If you withdraw in period 2, then you get d_2 units of the good.

  3.) If more than $\frac{N}{d_1}$ consumers want to withdraw in period 1, then funds will be distributed on a "first come, first served basis". This is sometimes called the sequential service constraint.

- The sequential service constraint arises from the fact that types are not verifiable. In particular, a Type 2 can always *pretend* to be a Type 1, and get d_1. (A Type 1 would never pretend to be a Type 2). A Type 2 might want to masquerade as a Type 1 if $d_1 > 1$, since in that case, if enough *other* Type 2's pretend to be Type 1's and withdraw early, the bank will not have sufficient funds to pay off the remaining Type 2's. Of course, it is only when $d_1 > 1$ that the bank is providing any liquidity insurance to the consumer, since a consumer can always guarantee himself $d_1 = 1$ by himself.

- Since the optimal decision of a Type 2 depends on what other Type 2's do, the problem is essentially *game theoretic*. As is often the case, this 'game' turns out to have multiple equilibria.

Optimal Contract

Let's first look for an equilibrium in which Type 2's believe other Type 2's will abide by the contract. The bank's (implicit) contract design problem is as follows:

$$\max_{x, d_1, d_2} \quad t \cdot U(d_1) + (1-t) \cdot U(d_2)$$

subject to

1.) $t \cdot N d_1 = x \cdot N$

2.) $(1-t) N d_2 = (1-x)(1+r) N$

3.) $d_1 \leq d_2$

Where x is the proportion of projects that are terminated early.

The first constraint is the resource constraint in period 1 (assuming no Type 2's show up early).

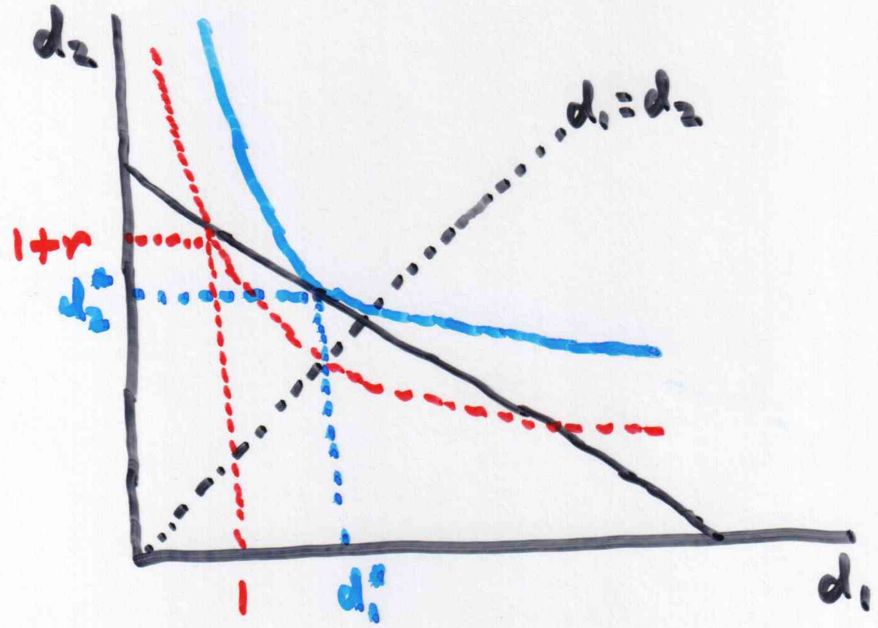
The second constraint is the resource constraint in period 2 (again assuming no early Type 2's).

The third constraint is an incentive constraint, which ensures that no single Type 2 consumer wants to withdraw early.

Note that we can substitute the first constraint into the second and get (after cancelling the N's):

$$d_2 = \frac{1+r}{1-t} - \frac{t\cdot(1+r)}{1-t} d_1$$

Now we have a problem very much like the usual problem of maximizing utility subject to a budget constraint. Its solution can be depicted in the following graph:



Notice that the autarky outcome $c_1 = 1$ and $c_2 = 1+r$ is feasible (i.e., it lies on the budget constraint). However, if consumers are sufficiently risk averse, the bank can construct a welfare improving contract, which allows consumers to reach the Blue indifference curve.

To characterize the optimal contract more precisely, sub in the constraint and we get the follow simple calculus problem (ignoring the incentive constraint for the moment).

$$\max_{d_1} \; t \cdot U(d_1) + (1-t) \cdot U\left[\frac{1+r}{1-t} - \frac{t(1+r)}{1-t} d_1\right]$$

Using the 'chain rule', we get the following first-order optimality condition:

$$\boxed{U'(d_1) = (1+r)U'(d_2)}$$

Notice that this guarantees that $d_2 > d_1$, so the incentive constraint is satisfied.

Suppose the utility function takes the form $U(c) = \frac{c^{1-\rho}}{1-\rho}$, where ρ measures "risk aversion". As $\rho\uparrow$, the consumer becomes more risk averse. Then $U'(c) = c^{-\rho}$ and we can write the first order condition as $\left(\frac{d_2}{d_1}\right) = (1+r)^{1/\rho}$. This equation along with the budget constraint determines d_1 + d_2 as functions of r, t, and ρ.

Numerical Example

Suppose $\rho = 2$. In this case, the FOC becomes:

$$\frac{d_2}{d_1} = \sqrt{1+r} \implies d_2 = \sqrt{1+r}\, d_1$$

Sub this into this constraint + collect terms in d_1

$$\left[\sqrt{1+r} + \frac{t(1+r)}{1-t}\right] d_1 = \frac{1+r}{1-t} \implies \boxed{d_1 = \frac{\sqrt{1+r}}{1-t+t\sqrt{1+r}}}$$

$$\implies \boxed{d_2 = \frac{1+r}{1-t+t\sqrt{1+r}}}$$

Now suppose $t = 1/4$ and $r = 1$. Then we get

$$d_1 = \frac{\sqrt{2}}{.75 + .25\sqrt{2}} \qquad d_2 = \frac{2}{.75+.25\sqrt{2}}$$

$$= 1.282 \qquad\qquad = 1.813$$

Even though $d_2 < 1+r$, the extra consumption you get in the event of a "liquidity draw" more than compensates you

Exercise 1: Show numerically that expected utility is greater with the contract than under autarky.

Exercise 2: When $\rho = 1$, then utility takes the form $U(c) = \ln(c)$. Show that in this case the optimal contract does not improve welfare.

Bank Runs

- The previous equilibrium depends on the belief of each Type 2 consumer that *other* Type 2 consumers will wait. Although it is optimal for a Type 2 to wait if other Type 2's wait, it is *not* optimal to wait if you think a sufficiently large number of other Type 2's won't.

- To see why, note that since N is large, the "law of large numbers" means that $t \cdot N$ Type 1's will withdraw in period 1. (Type 1's always withdraw). This means that if the bank liquidates *all* its projects, the most that can be made available for Type 2's in period 1 is $N - t \cdot N d_1 = N(1 - t d_1)$. Now, since there are $(1-t)N$ Type 2's, and $d_1 > 1$, the available funds is *less* than the potential demands of the Type 2's. In particular, as long as at least $\frac{1 - t d_1}{1 - t}$ Type 2's show up early, there won't be anything left for those who wait, and it will be optimal to run to the bank. For example, in the above numerical example, it is optimal to run if you think that there is at least a $\frac{1 - .25(1.28)}{.75} = 91\%$ chance that others will run!

Deposit Insurance

- The bank run equilibrium is very inefficient. It is a worse outcome than if there were no banks at all! No productive long-term investment projects are completed, and due to the sequential service constraint, it is possible that some desperate Type 1 consumers will be unable to access their money.

- Does this mean we should not have banks? It turns out that there is a simple solution to the problem. The only reason Type 2's run to the bank is the belief that they might not get their money back. Suppose an outside agent, the govt., simply announces that they are guaranteeing all the deposits. In this case, no Type 2 ever runs to the bank, even if he thinks others will! As a result, the govt. never has to make good on its promise to bail out the bank. The guarantee is costless to the taxpayer!

Moral Hazard

- Something is wrong here! Why has the govt. been bailing out so many banks recently?
- The problem with deposit insurance, like any insurance, is that it creates incentives for additional risk taking. From a banker's perspective, deposit insurance means it's "heads I win, tails you lose". The recent banking crisis was caused by excessive risk taking on the part of banks. (This is not really captured in the above model, since the riskiness of the project is fixed).
- In theory, deposit insurance is supposed to be accompanied by regulatory oversight, which audits banks' portfolios and prevents them from taking on excessive risks. In this case, regulators were "asleep at the switch"!
- Although it's hard to be sympathic with a bunch of rich bankers, you can't really blame them for the recent crisis. Banking is a competitive business. If your rivals are allowed to take on more risks, then you'd better take on more too. Otherwise you'll lose your customers.