# Topics for Today

1.) The Production Possibilities Frontier

2.) Competitive Equilibrium

3.) Pareto Optimality

4.) The Relationship Between Competitive Equilibria and Pareto Optima
   - The First + Second Welfare Theorems

5.) Computing Competitive Equilibria by Solving a Social Planning Problem

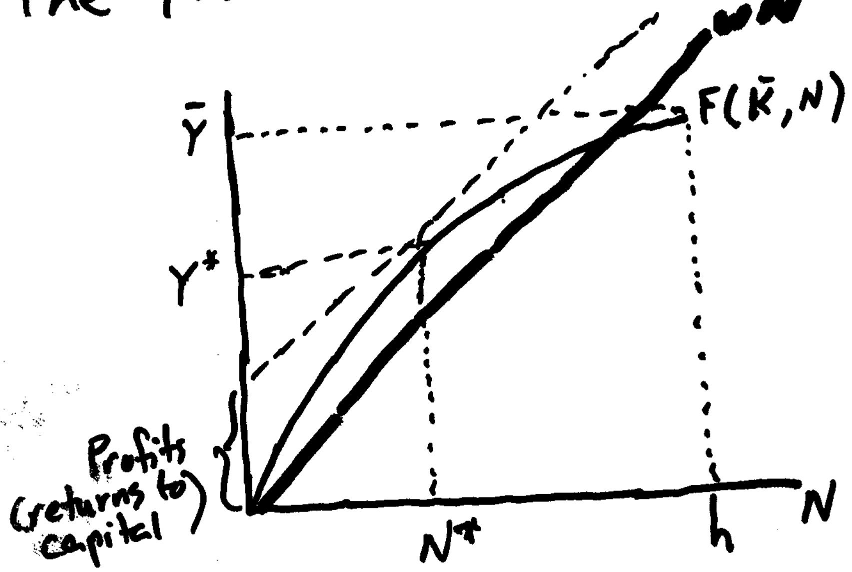
6.) Comparative Statics

7.) An Algebraic Example

# Production Possibilities Frontier

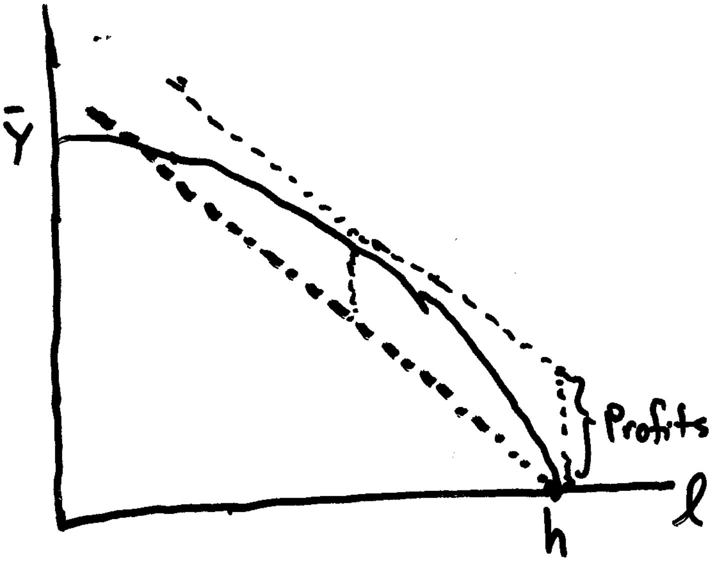
- The Production Possibilities Frontier (PPF) describes the menu of choices available to society given resources and the state of technology.

- When consumption + leisure are the only two goods, the PPF is a simple transformation of the production function.



Production Function

$$Y = F(N)$$



Production Possibilities Frontier

$$Y = F(h - \ell)$$

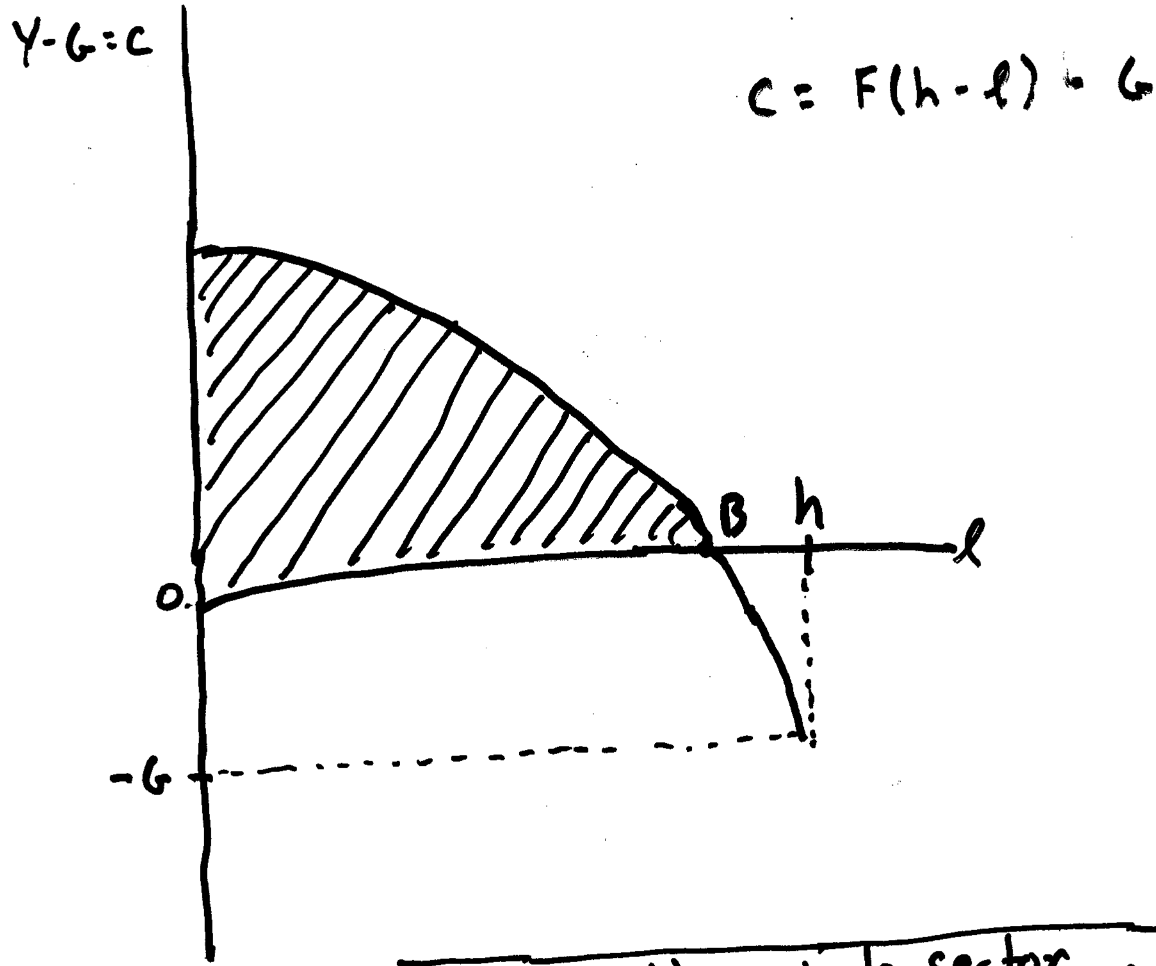
$$\frac{dY}{d\ell} = -F_N$$

∨

The output cost of an extra unit of leisure is given by the Marginal Product of Labor

Now Suppose the Government consumes G units of output, and pays for its purchases by levying (lump-sum) taxes on the private sector. [With only one-period, the govt. must balance its budget, so $T = G$].

This shifts the privcte sector's PPF down by G.



Now the private sector cannot consume h units of leisure. It must work at least h - B in order to produce the goods the govt. ~~by~~ buys

# Competitive Equilibrium

Where on the PPF will the economy locate?

If the economy is *competitive* (and there are no market imperfections), the economy will locate at a point of mutual tangency between an Indifference Curve and the PPF. The slope of the common point of tangency is given by (minus) the wage rate. Market prices are an "*invisible hand*", reconciling the choices of firms & households.

In a competitive equilibrium, 3 thing are true:

1.) Households are maximizing utility subject to their budget constraints

2.) Firms are maximizing profits subject to theire resource & technology constraints

3.) Markets Clear - $N^s = N^d$

$$C + G = Y$$

(of course, the govt. budget constraint must hold too, but since the govt. is exog., we can ignore this)

In our 2 good economy, a competitive equilibrium can be described by 7 equations

$$\left.\begin{aligned}
&1.)\quad \frac{U_l}{U_c} = w \\
&2.)\quad C = wN^s + \pi - T \\
&3.)\quad h = l + N^s
\end{aligned}\right\}\ \text{Households}$$

$$4.)\quad zF_N(\bar{K},N^d) = w \quad \} \text{ Firms}$$

$$5.)\quad N^s = N^d \quad \} \text{ Market Clearing}$$

$$6.)\quad \pi = zF(\bar{K},N^d) - wN^d \quad \} \text{ Definition of Profits}$$

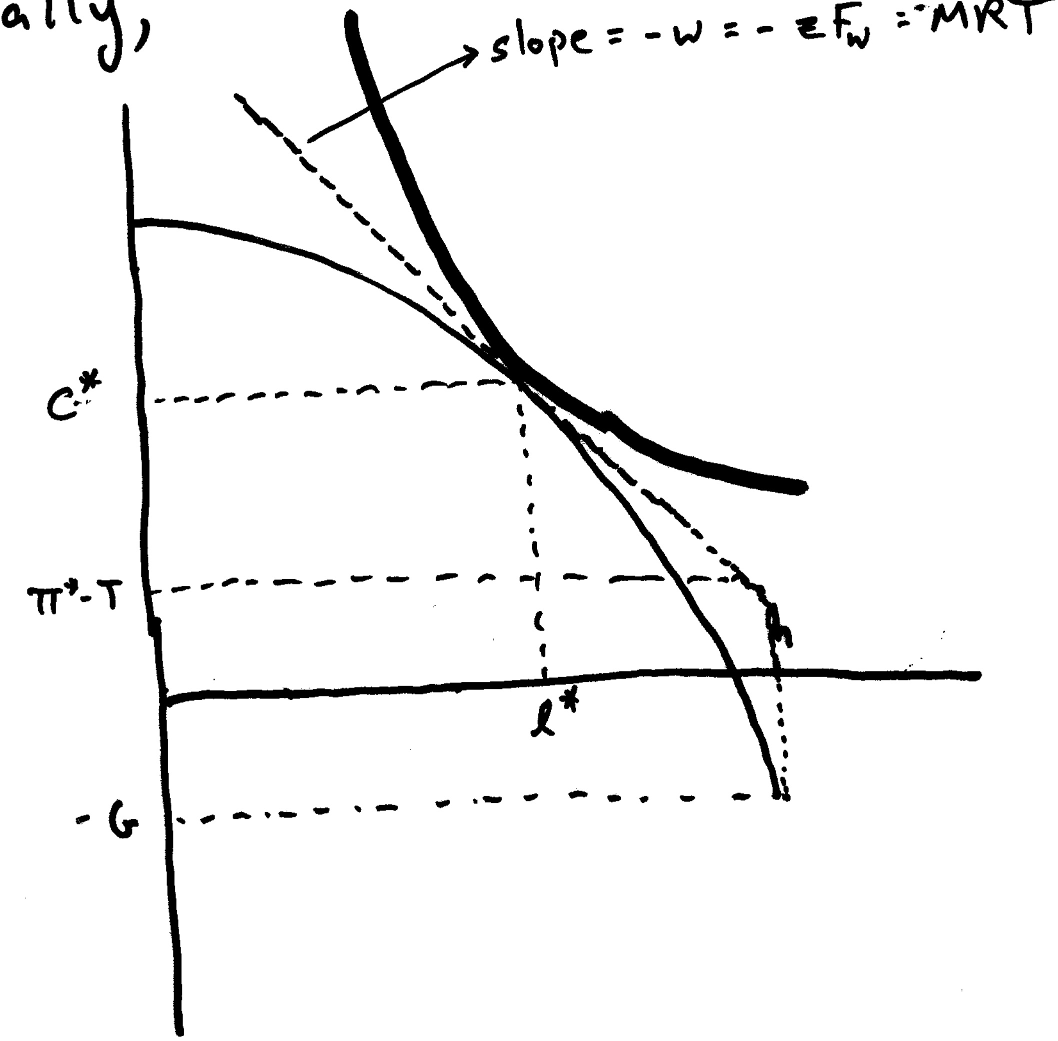
$$7.)\quad T = G \quad \} \text{ Govt. Budget Constraint}$$

The 7 equations determine $(c, l, N^s, N^d, w, \pi, T)$ as functions of the exogenous variables $(G, z, \bar{K})$.

These 7 equations can be condensed to the following 3 equations:

$$\left.\begin{array}{ll} 1.) & \dfrac{U_\ell(C, h-N)}{U_c(C, h-N)} = w \\ 2.) & zF_N(\bar{K}, N) = w \\ 3.) & C + G = zF(\bar{K}, N) \end{array}\right\} \Longrightarrow (C, N, w)$$

Graphically,



Note, at a competitive equilibrium

$$\boxed{MRS = w = MPL = MRT}$$

↓

~~Abili~~
Willingness
to substitute

↓

Ability to
substitute.

# Pareto Optimality

- What are the *normative* implications of competitive Equilibria? Are competitive equilibria "efficient"? Are they "fair"?

- These are among the oldest (and most important) questions in all of Economics. Adam Smith answered them in the affirmative, but was rather vague in defining "efficiency" & "fairness".

- In modern economics, an outcome is *defined* to be efficient if it is *Pareto Optimal.*

- Pareto Optimality = Can't make one person better off without making someone else worse off.

- Note, this is a very weak notion of efficiency, since it ignores the *distribution* of goods!

- In our one agent/representative household economy, Pareto Optimality just means you are on the PPF at the highest Indifference Curve.

Under certain conditions, there is a close connection between Competitive Equilibria and Pareto Optima. This connection is described by the following 2 Theorems:

**First Welfare Theorem**: As long as markets are complete and everyone behaves competitively (i.e., no single agent, or collusive group of agents, can influence prices) then a competitive equilibrium is Pareto Optimal.

Note, the assumption that markets are complete effectively rules out (non-pecuniary) **externalities**.

**Second Welfare Theorem**: As long as markets are complete and everyone behaves competitively, and preference + production sets are convex, then any Pareto Optimal outcome can be achieved as a competitive equilibrium if appropriate (lump-sum) redistributions of wealth are arranged ahead of time.

- If our economy satifies these conditions, then the 2nd. Welfare Theorem offers us a short-cut to computing a Competitive Equilibrium.

- Rather than look for a set of prices that simultaneously clear all markets, we can ignore prices & markets altogether, and simply solve a "Social Planner's Problem", i.e., Maximize the Utility of the Households subject to the economy's resource & technology constraints. In other words, we don't have to worry about firms at all!

- If for some reason we need to know what market prices are, we can simply substitute the Pareto Optimal quantities into the appropriate marginal conditions characterizing prices.

# Example

## Competitive Equilibrium

$$\left.\begin{array}{l} 1.)\ \dfrac{U_\ell}{U_c} = w \\[2ex] 2.)\ zF_N(K,N) = w \\[2ex] 3.)\ C + G = Y = zF(K,N) \end{array}\right\} \ell, N, C, w$$

## Pareto Optimum

$$\max_{c,\ell} U(c,\ell) \quad \text{subject to} \quad C = zF(K,h-\ell) - G$$

$$\mathcal{L} = U(c,\ell) + \lambda\left[zF(K,h-\ell) - C - G\right]$$

FOCs

$$\left.\begin{array}{ll} c: & U_c - \lambda = 0 \\ \ell: & U_\ell - \lambda z F_N = 0 \\ \lambda: & zF(K,N) - C - G = 0 \end{array}\right\} \text{These produce the same solution for } (C,N) \text{ as the above equations.}$$

# An Algebraic Example

Suppose $U(c, \ell) = \ln C + \beta \ln \ell$

$zF(K, N) = zK^{\theta}N^{1-\theta}$

Goal: Find expressions for $C(z, K, G)$, $N(z, K, G)$ and $w(z, K, G)$

1.) $\frac{U_{\ell}}{U_{c}} = \frac{\beta C}{h - N} = w$

2.) $(1-\theta) zK^{\theta}N^{-\theta} = (1-\theta)\frac{Y}{N} = w$

3.) $C + G = Y$

4.) $zK^{\theta}N^{1-\theta} = Y$

To simplify the algebra, suppose $G = \lambda Y$

Then, from (3), $C = (1-\lambda) Y$

Use this to sub out $C$ from (1).

Now get 3 simulteneous eqs.)

1.)' $\beta(1-\lambda)Y = w(h-N)$

2.)' $(1-\theta)Y = wN$

4.) $Y = zK^{\theta}N^{1-\theta}$

Sub (2)' into (1)' and solve for $N$

$$\boxed{N = \left[\frac{1-\theta}{\beta(1-\lambda)+(1-\theta)}\right] h}$$

Note: $\beta\uparrow \Longrightarrow N\downarrow$ } Household values leisure more, so works less

$\lambda\uparrow \Longrightarrow N\uparrow$ } Negative income effect from higher govt. spending & taxes

Next, sub in the expression for $N$ and (4) into (2.)', and solve for $w$

$$\boxed{w = (1-\theta)\,Y/N = (1-\theta)\,z\,K^{\theta} h^{-\theta}\left[\frac{1-\theta+\beta(1-\lambda)}{1-\theta}\right]^{\theta}}$$

Note: $\beta \uparrow \implies w \uparrow$

$\lambda \uparrow \implies w \downarrow$

$z \uparrow \implies w \uparrow$

We can interpret these results by deriving Labor Supply + Labor Demand Curves

From the firm's FOC

$$(1-\theta) z K^{\theta} N^{d-\theta} = w$$

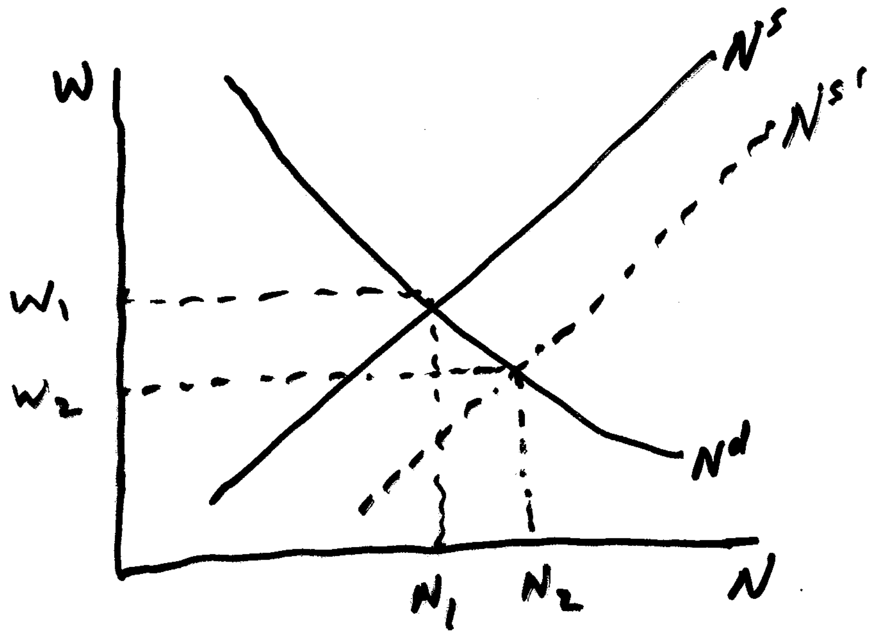
$$\Longrightarrow \boxed{N^{d} = \left[\frac{(1-\theta) z K^{\theta}}{w}\right]^{1/\theta}}$$

From the household's FOC

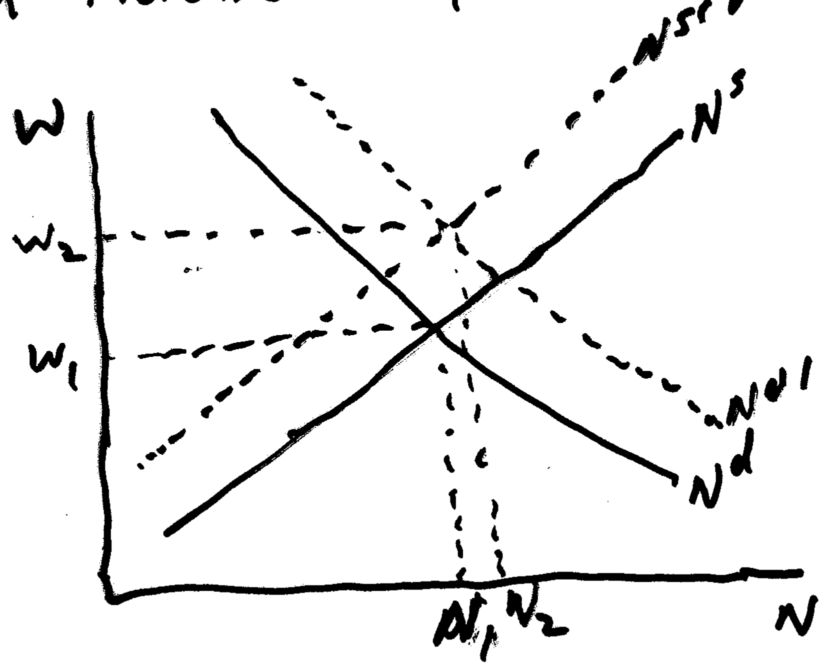
$$\beta C = \beta(1-\lambda)Y = \beta(1-\lambda) z K^{\theta}(N^{s})^{1-\theta} = w(h-N^{s})$$

$$\Longrightarrow \boxed{\frac{(N^{s})^{1-\theta}}{h-N^{s}} = \frac{w}{\beta(1-\lambda) z K^{\theta}}}$$

1.) An increase in G ( $\lambda\uparrow$ )



2.) An increase in productivity ( $z\uparrow$ )



Note: $w\uparrow$ unambiguously
Effect on N is ambiguous
(offsetting income & substitution effects)