SIMON FRASER UNIVERSITY
Department of Economics

Econ 305
Intermediate Macroeconomic Theory

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PROBLEM SET 1
(Solutions)

1. The household's optimal consumption/leisure choice is characterized by the equality between the (real) wage rate and the Marginal Rate of Substitution between consumption and leisure:

$$\frac{U_\ell}{U_c} = w$$

The labor supply curve just expresses this as a relationship between labor supply and the wage, for a *given* level of consumption/income. Taking partial derivatives, and substituting the time constraint, $h = \ell + N^s$, gives us

$$\frac{\alpha C}{\ell} = w \qquad \Rightarrow \qquad N^s = h - \frac{\alpha C}{w}$$

Notice that N^s is an increasing function of w. Also notice that when C increases, N^s decreases, (ie., the labor supply schedule shifts left) due to a negative income effect. The same thing happens when α increases, since now the household places a higher (relative) value on leisure.

The firm's optimal labor input decision is characterized by the equality between the (real) wage and the marginal product of labor:

$$(1-\theta)zK^\theta (N^d)^{-\theta} = w$$

The labor demand curve just expresses this as a relationship between labor demand and the wage, for *given* values of z and K. Solving for N^d gives

$$N^d = \left(\frac{(1-\theta)zK^\theta}{w}\right)^{1/\theta}$$

Notice that N^d is a decreasing function of w. It shifts right when either z or K increase.

2. To compute a competitive equilibrium, we just need to add the economy's aggregate resource constraint to the household's and firm's optimality conditions. With the new utility function the household's optimality condition becomes:

$$\beta C = w$$

(Notice that in this case the labor supply curve is horizontal at $w = \beta C$. This is because the marginal utility of leisure is constant). The firm's optimality condition continues to be

$$(1-\theta)zK^{\theta}N^{-\theta} = w$$

The aggregate resource constraint is just

$$C + G = Y = zK^{\theta}N^{1-\theta}$$

Since by assumption $G = \lambda Y$, we can use the resource constraint to eliminate C and get the folowing 2-equation system:

$$\begin{aligned} \beta(1-\lambda)[zK^{\theta}N^{1-\theta}] &= w \\ (1-\theta)zK^{\theta}N^{-\theta} &= w \end{aligned}$$

Solving this system produces the competitive equilibrium values of w and N. Solving for N first gives

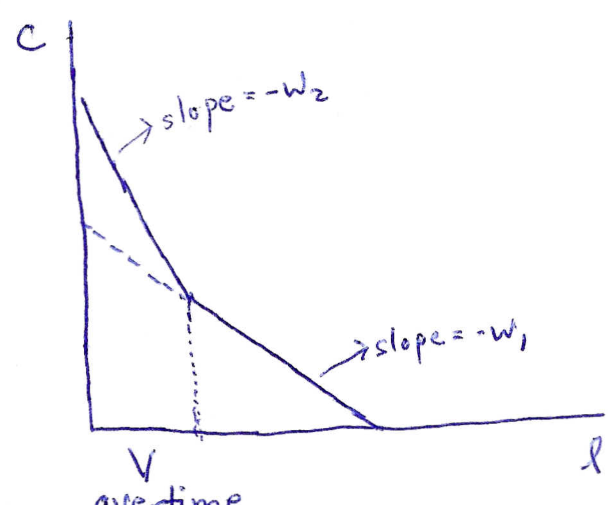
$$N = \frac{1-\theta}{\beta(1-\lambda)}$$

Substitute this back into either equation and get the equilibrium value of w

$$w = (1-\theta)zK^{\theta}\left[\frac{\beta(1-\lambda)}{1-\theta}\right]^{\theta}$$

Given N and w it is straightforward to compute Y and C. Notice that when government spending increases as a fraction of ouptut (i.e., $\lambda \uparrow$), the equilibrium value of N increases and the equilibrium value of w decreases. This is because the negative income effect shifts out/down the labor supply curve. On the other hand, notice that when productivity increases (i.e., $z \uparrow$), wages rise but employment stays fixed. The increase in z shifts out the labor demand curve, but shifts up the labor supply curve (because Y and C increase). Employment remains constant since the income effect exactly offsets the substitution effect.

3. **(a)** A higher wage rate for overtime produces a 'kink' in the household's budget constraint as follows



(b) It is visually obvious that as long as the household's Indifference Curve is convex (and differentiable), it will never be optimal to select the kink point. The marginal rate of substitution cannot simultaneously be equal to two different wage rates! The only case where the household might choose the kink point is when the marginal rate of substitution is itself discontinuous, (ie., when the Indifference Curve also has a kink).

(c) When just the overtime wage increases, the substitution and income effects depend on the initial position of the household. If its original tangency is far away from the overtime point, the overtime wage increase may be irrelevant. (Suppose the overtime wage only occurred for hours worked in excess of 70 hours a week. This would be irrelevant for most people!). Of course, if you are already working overtime then the increase has the usual income and substitution effects. More interestingly, for those households close to the kink, the overtime pay increase may induce them to start working overtime.

4. Case 1: Production/Sales Tax The firm's after-tax profits become

$$\pi = (1-\tau)zK^{\theta}N^{1-\theta} - wN$$

The profit maximizing first-order condition becomes:

$$(1-\tau)(1-\theta)zK^{\theta}N^{-\theta} = w$$

Since the tax reduces the after-tax marginal product of labor, it effectively shifts the labor demand curve to the left. For a given labor supply curve, this will reduce employment and wages.

Case 2: Profits Tax Now the firm's after-tax profits are:

$$\pi = (1-\tau)[zK^{\theta}N^{1-\theta} - wN]$$

The profit maximizing first-order condition becomes:

$$(1-\tau)(1-\theta)zK^{\theta}N^{-\theta} = (1-\tau)w$$

Notice that the $(1-\tau)$ cancels from both sides, and so the tax becomes irrelevant. It does not influence the firm's hiring decisions. Intuitively, the tax lowers the marginal product *and* the marginal cost of labor by the same amount. The firm may not be happy to have its profits taxed, but given the tax, the best thing it can due is to continue to maximize profits as before.

5. We now have an extended Cobb-Douglas production function,

$$Y = K^{1/3}L^{1/3}H^{1/3}$$

where the labor input has been subdivided into a skilled component ('human capital', H) and an unskilled component ('Labor', L).

(a)

$$MPL = (1/3)K^{1/3}L^{-2/3}H^{1/3}$$

Notice that an increase in H increases the marginal product of labor. That is, having skilled workers around enhances the productivity of unskilled workers.

(b)

$$MPH = (1/3)K^{1/3}L^{1/3}H^{-2/3}$$

Notice that an increase in H decreases the marginal product of human capital. This just reflects diminishing returns. (Is that Ph.D. really worth it?)

(c) This is the only tricky part of the question. Note that a skilled worker not only earns the return to human capital, but she also receives the return to raw labor as well. (Presumably, once you get a college degree, you don't forget how to do routine tasks!). Thus, if we let W_s denote the skilled wage, we have:

$$W_s = MPL + MPH$$

(d) Now letting W_u denote the unskilled wage, we get the following expression for the wage premium for skilled labor:

$$\frac{W_s}{W_u} = \frac{MPL + MPH}{MPL} = 1 + \frac{MPH}{MPL} = 1 + \frac{L}{H}$$

Thus, increases in human capital reduce the skill premium. Having more college graduates around not only bids down wages for jobs needing a college education, it also tends to raise the wages of unskilled workers as well, since their productivity rises. For both reasons the skill premium falls with increases in H.

(e) These answers suggest that college scholarships equalize the income distribution.