SIMON FRASER UNIVERSITY
Department of Economics

Econ 305 | Prof. Kasa
Intermediate Macroeconomic Theory | Spring 2011

# PROBLEM SET 2
(Solutions)

1. Things like 'productivity' and 'capital' are hard to measure. In practice, economists often measure them indirectly, using economic theory. This question asks you to this. Suppose an economy is described by the Solow growth model. In particular, suppose the production function is $Y = AK^{1/2}L^{1/2}$, where $A$ represents the level of total factor productivity. Our job is to figure out what $A$ is, based on observed data. For simplicity, suppose that the level of $A$ is known to be constant. Here's what else we know: (1) population (and the labor force) grows at the annual rate of 3% (i.e., $n = .03$), (2) Capital depreciates at the annual rate of 7% (i.e., $\delta = .07$), (3) The economy has converged to a steady state, with a per capita income of $20,000, and (4) The saving/investment rate is 20% (i.e., $s = .20$).

   **(a)** Derive an expression for the steady state capital/labor ratio in terms of $A$, $s$, $n$, and $\delta$.

   *The steady state is characterized by the equality, $sy = (n + \delta)k$, where $y$ is per capita output, and $k$ is the capital/labor ratio. Using the above production function, we can write this as,*

   $$sAk^{1/2} = (n + \delta)k$$

   *Solving for $k$ gives the following expression for the steady state capital/labor ratio, $k^*$*

   $$k^* = \left(\frac{sA}{n+\delta}\right)^2$$

   **(b)** Using your answer to part (a), derive an expression for the steady state level of per capita income.

   *Once we know $k^*$ it is easy to solve for steady state output, $y^*$. It is just $y^* = A\sqrt{k^*}$. Using the answer to part (a), we get*

   $$y^* = A\left(\frac{sA}{n+\delta}\right) = A^2\left(\frac{s}{n+\delta}\right)$$

   **(c)** Using your answer to part (b) and the data you observe, calculate what $A$ is.

   *Using the answer to part (b) we get the following expression for $A$ in terms of the data we know*

   $$A^2 = y^*\left(\frac{n+\delta}{s}\right) \qquad \Rightarrow \qquad A = \sqrt{y^*}\sqrt{\frac{n+\delta}{s}}$$

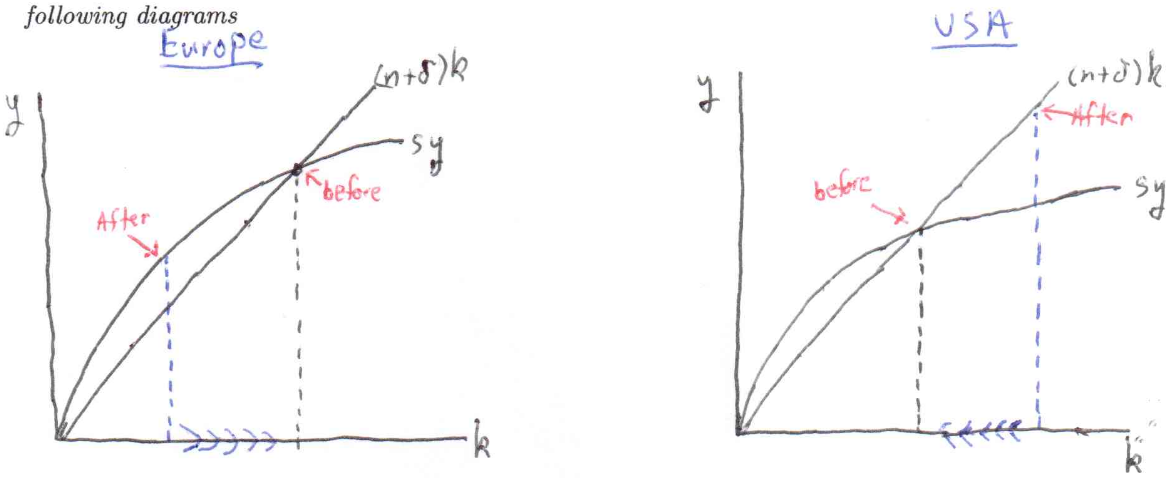
   *Substituting in the given information, we get*

   $$A = \sqrt{\frac{(20,000)(.03 + .07)}{.2}} = \sqrt{10,000} = 100$$

2. For the sake of argument, suppose World War II had the following contrasting effects on Europe and the United States: Europe suffered a massive destruction of its capital stock relative to its population, while the U.S. suffered a large loss of its population relative to its capital stock. That is, $K/L$ suddenly fell in Europe, while $K/L$ suddenly rose in the U.S. For simplicity, suppose these were the only things that changed (i.e., saving rates, population growth, production functions, etc., stayed the same).

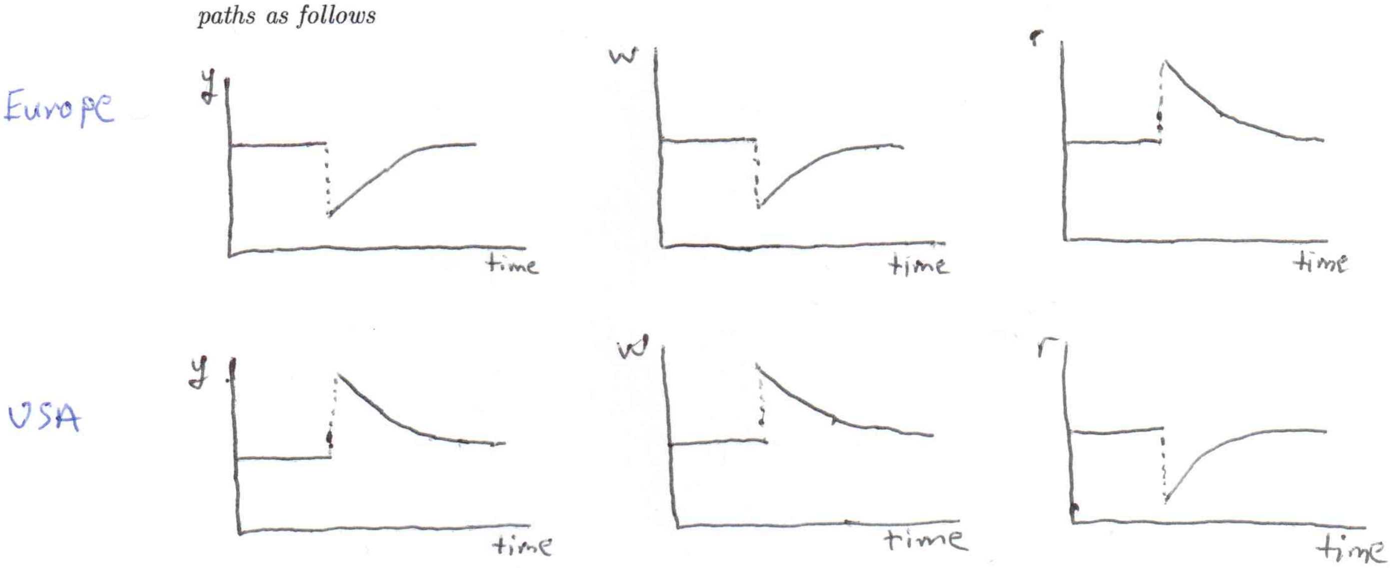
**(a)** Illustrate the effects of the war on the U.S. and Europe in a 'Solow diagram' (i.e., with the capital/labor ratio on the horizontal axis, and output and saving on the vertical axis).

*Assuming they were in a steady state before the war, we can visualize the effects the war in the following diagrams*




**(b)** Compare and contrast how the U.S. and Europe would recover after the war. Describe the time paths of per capita output, the wage rate, and the interest rate. Who grows faster after the war?

*By assumption, each economy's steady state has not changed, since the production function is the same, population and depreciation rates are the same, and the saving rate is the same. Therefore, over time, each economy gradually converges back to its original steady state. In the U.S., the capital labor ratio falls. In Europe, the capital/labor ratio rises. Assuming labor and capital are paid their marginal products (as they would be in competitive factor markets), this means that wages fall over time in the U.S., while they rise over time in Europe (since the marginal product of labor is an increasing function of $k$). On the other hand, because of diminishing returns, as $k$ falls in the U.S., interest rates rise over time, while in Europe they fall. We can depict the time paths as follows*

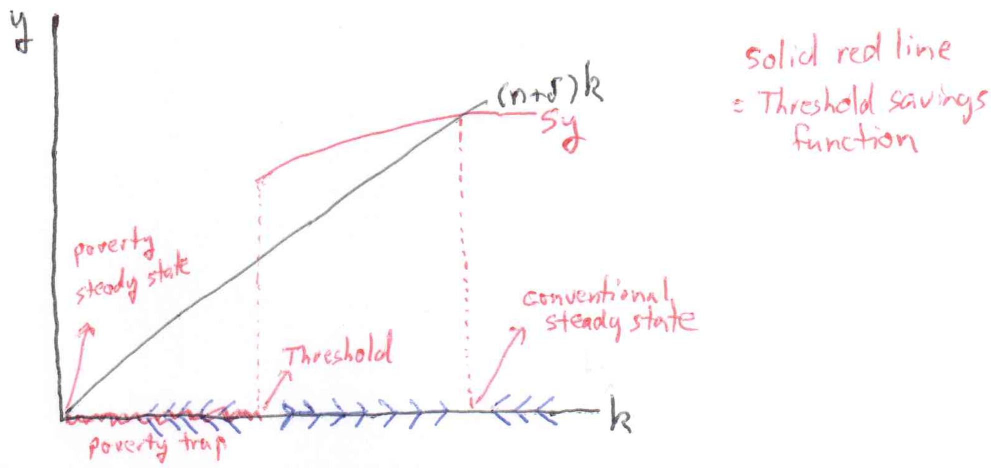



*In some respects, this resembles what happened after the war, at least qualitatively. Japan and Europe started with relatively low $k$, $y$, and $w$, and then gradually caught up with the U.S.*

3. In class, we assumed that people saved a fixed fraction of their income, no matter what the level of their income was. This might not be such a good assumption at very low levels of income. Instead, suppose that at very low levels of income, people are simply worried about subsistence, and consume all their income, i.e., the saving rate is zero. For simplicity, suppose that once this subsistence level is exceeded, people start saving a fixed fraction of their income, as we assumed before.

**(a)** Illustrate this new savings function in the usual 'Solow diagram', with the capital/labor ratio on the horizontal axis and per capita income on the vertical axis.

*If people cannot afford to save anything until some threshold level of income (and capital) is reached, we get the following modified Solow diagram*




**(b)** Does the economy have a unique steady state? Does the long run fate of this economy depend on where it starts?

*With this savings function, the economy now has multiple steady states, and where it ends up depends crucially on where it starts. If the economy begins below the threshold, it does not save enough to offset depreciation and population growth, and the economy actually shrinks. In this simple example, it shrinks all the way to zero, but we could easily modify it so that it would not go all the way to zero. On the other hand, if the economy is initially prosperous enough, saving is high enough to actually increase the capital stock, and the economy grows. (Note, although the zero steady state is also a steady state with the conventional savings function, since with no capital there is no output or saving, it is not stable. That is, even a tiny, infinitesimal, amount of initial capital will send you to the high output steady state. In contrast, with a threshold savings function, the low output steady state becomes stable. That's the key difference here.*

**(c)** Are there any policy implications from this analysis?

*Many development economists have argued that very poor countries are described by this sort of picture. It is often called a 'development trap'. Given this, the policy recommendation is clear - somehow get people to save more, presumably by giving government assistance to very poor households. The idea is to 'jump start' the market and ignite the process of capital accumulation. A more modern version can be seen in the recent popularity of 'micro finance', which gives people better access to credit. Although this might seem to be different, since you're trying to relax borrowing constraints, the idea here is that people would like to borrow in order to invest, not consume. It's a little harder to depict this story, however, since you really need two kinds of production functions, one that is low-yielding and does not require credit and large up-front investment, and another higher-yielding production function that can only be exploited if people have access to credit.*