SIMON FRASER UNIVERSITY

Department of Economics

Econ 305   Prof. Kasa

Intermediate Macroeconomic Theory   Spring 2012

PROBLEM SET 1

(Due February 10)

1. (20 points). The following table contains recent data from the Penn World Tables (note all GDPs expressed relative to the USA):

Country	Per Capita GDP (2000)	Investment/Saving Rate
USA	1.0	.245
France	0.691	.259
Japan	0.713	.361
S. Korea	0.556	.439
Mexico	0.261	.235
China	0.183	.288
Uganda	0.027	.042

Assume that each country is described by the Solow model, with production function $Y = AK^{1/3}L^{2/3}$, and assume that each country is in a steady state. Assume that capital depreciation and labor force growth are identical across countries. (Note, you will need a calculator for this question).

(a) Assuming that productivity, A, is identical across countries, use the above data on investment rates to calculate the predicted GDP of each country (relative to the USA). How do the predictions match up to the actual data given in the first column?

(b) Use the model to infer what productivity must be in each country (relative to the USA). Any surprises? According to this model, which is more important in explaining cross-country income differences, investment or productivity?

(c) Briefly describe at least two caveats to this kind of analysis (besides the obvious one that the Solow model might be wrong!).

2. (25 points). Consider a standard Solow model with two modifications: (1) Assume the production function takes the "AK" form, $Y = AK$. (As discussed in class, this might be a good approximation under a broad view of capital and/or if there are externalities associated with capital accumulation), (2) Instead of assuming that population growth is constant, assume it depends on the level of per capita income. In particular, assume that $n = n_0 \cdot k^\gamma$, where $k = K/L$ is the capital/labor ratio, n_0 is a constant, and $\gamma > -1$ is a fixed parameter. As usual, assume the saving rate, s, is constant, and that depreciation is a constant fraction, δ, of the existing capital stock.

(a) Following the usual steps, derive an expression for the steady state capital/labor ratio. Use a graph to illustrate how the economy evolves over time. (Hint: Put k on the horizontal axis, and per capita output and investment on the vertical axis). Under what conditions on γ is the steady state stable? Explain intuitively.

(b) How does per capita income respond to an increase in the saving rate, s? How does your answer depend on γ? Explain.

3. (25 points). Consider a standard one-sector Solow model with a fixed savings rate s. Output is produced via the production function

$$Y_t = K_t^\alpha L_t^{1-\alpha}$$

As usual, labor is inelastically supplied and grows at the exogenous rate n (i.e., $L_{t+1} = (1+n)L_t$).

Suppose that each unit of output produced generates Ω_t units of 'pollution', and that due to exogenous technological progress in pollution abatement, Ω_t decreases over time at rate g_a (i.e., $\Omega_{t+1}/\Omega_t = 1/(1+g_a)$). In addition, suppose that there is an 'abatement technology' that allows resources to be diverted into pollution reduction. Specifically, if θ represents the share of output used in pollution reduction, then net pollution emission, E_t, is given by

$$E_t = a(\theta)\Omega_t Y_t \tag{1}$$

where $a(\theta)$ is assumed to be a positive, decreasing function. For simplicity, assume that θ is constant and exogenous.

As usual, for notational convenience, let y_t represent net output available for consumption and capital accumulation per capita. That is,

$$y = \frac{(1-\theta)Y}{L}$$

Similarly, let k and e be capital and net pollution emission per capita. Using this notation, we have the following 'green Solow' model:

$$
\begin{aligned}
y_t &= (1-\theta)k_t^\alpha \\
\Delta k_{t+1} &= s(1-\theta)k_t^\alpha - (\delta+n)k_t \tag{2} \\
e_t &= a(\theta)\Omega_t k_t^\alpha \tag{3}
\end{aligned}
$$

where δ is the depreciation rate of capital. Evidently, from equation (2), the economy will converge to a unique steady state $k_t = k^*$. From equation (3), it is then clear that in the steady state (ie, when k_t is constant), pollution grows at a constant rate, g_E, given by

$$g_E = \frac{\Delta E_{t+1}}{E_t} = \frac{1+n}{1+g_a} - 1 \approx n - g_a$$

Of course, during the *transition* to the steady state, pollution may be growing either faster or slower than this. Now, let's <u>define</u> 'sustainable' growth to be a situation where $g_a \geq n$. That is, pollution remains bounded.

(a) It is often claimed that the time path of pollution within economies follows a so-called 'Environmental Kuznets Curve' (EKC), with pollution rising as the economy develops, and then eventually falling once the economy becomes wealthy enough (i.e., it traces out an inverted U-shape when plotted against either time or per capita income). Consider a sustainable economy, where $g_E < 0$. Under what conditions will this economy feature an EKC? (Hints: (1) Derive expressions for $\Delta k_{t+1}/k_t$ and $\Delta E_{t+1}/E_t$ as functions of k_t during the transition to the steady state, (2) The growth rate of $\Omega_t k_t^\alpha$ can be approximated by $\frac{\Delta\Omega_{t+1}}{\Omega_t} + \alpha\frac{\Delta k_{t+1}}{k_t}$). If you can't provide explicit analytical conditions, then at least try to explain intuitively how this relationship could arise.

(b) How does an increase in abatement effort (i.e., an increase in θ) affect the time path of pollution? Explain intuitively, and relate your conclusions to how a standard Solow model reacts to an increase in the savings rate. (Hint: You do not need to solve for anything. Just sketch out a time path).