Econ 305   Prof. Kasa
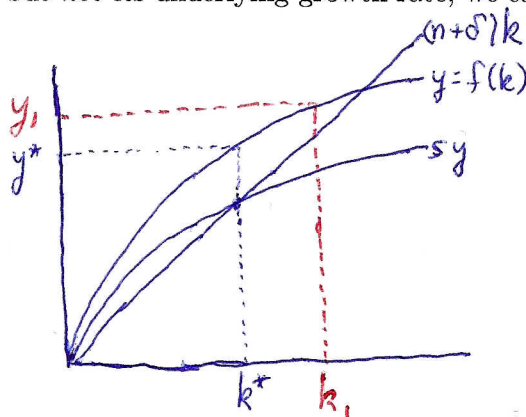Intermediate Macroeconomic Theory   Spring 2009

PROBLEM SET 2
(Answers)

1. If the economy is initially in a steady state, and a plague strikes, which suddenly reduces the size of the population, but *not* its underlying growth rate, we can visualize what happens in the following graph:


The capital labor ratio rises from k^* to k_1 (because the denominator falls). Per capita output rises from y^* to y_1. Due to diminishing returns, the higher capital/labor ratio depresses interest rates (which depend on the marginal produce of capital), but at the same time real wages rise (because more capital per worker raises the marginal product of labor). Over time the economy converges back to its original steady state, since by assumption, neither productivity, the saving rate, the depreciation rate, nor the population growth rate, have changed. (Note: These predictions are roughly consistent with the data from England that I presented in Lecture 6 (Part B)).

2. **(a)** The per capita production function can be expressed as follows:

$$y = (1-u)^{1-\theta} k^\theta$$

where $y = K/N$ and $k = K/N$. Notice that the term $(1-u)^{1-\theta}$ plays the role of 'total factor productivity' in conventional specifications.

(b) A reduction in u has the same effects as an increase in total factor productivity. Its effects can be visualized in the following graph:
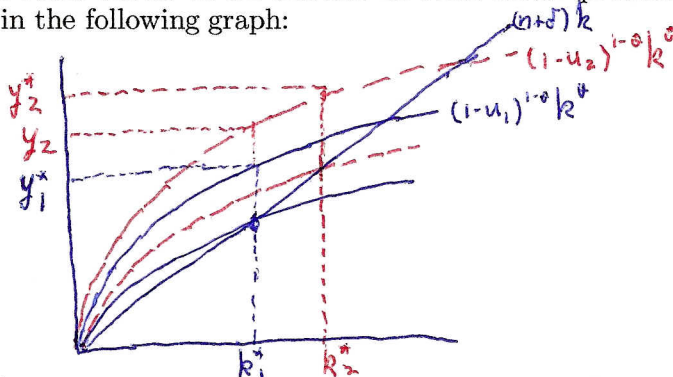
Steady state (per capita) output rises from y_1^* to y_2^*. The immediate short-run effect is for output to rise from y_1^* to y_2. This is less than the long run response because the reduction in u triggers additional investment, which raises the capital stock. The increased capital stock produces an additional boost to output.

3. **(a)** From the steady-state condition, $sy = (\delta + n + g)k$ we know:

$$\begin{aligned} s &= (\delta + n + g)(k/y) \\ &= (.04 + .03)(2.5) \\ &= 0.175 \end{aligned}$$

(b) Since the production function is Cobb-Douglas, we know that capital's share in national income is $\alpha = (MPK \cdot K)/Y$. Re-arranging this we have:

$$\begin{aligned} MPK &= \frac{\alpha}{(K/Y)} \\ &= \frac{0.3}{2.5} \\ &= 0.12 \end{aligned}$$

(c) The following condition characterizes the Golden Rule: $MPK = (\delta + n + g)$. Plugging-in the given data, we find that at the Golden Rule $MPK = .03 + .04 = .07$. Note that this is lower than the initial value of .12. Therefore to reach the Golden Rule steady state we need to increase k (which, due to diminishing returns, would reduce the MPK).

(d) First, note that we can compute the Golden Rule level of the capital/output ratio from the relation,

$$\begin{aligned} \frac{K}{Y} &= \frac{\alpha}{MPK} \\ &= \frac{.3}{.07} \\ &= 4.29 \end{aligned}$$

Then, just use the previous condition that,

$$\begin{aligned} s &= (\delta + n + g)(k/y) \\ &= (.04 + .03)(4.29) \\ &= 0.30 \end{aligned}$$

(Note, $(k/y) = (K/Y)$). Thus, to reach the Golden Rule, this economy will have to increase its savings rate from 17.5% to about 30.0%.

4. **(a)** We can write the per capita production function as $y = Ak^{\alpha}$. Use this in the steady state condition, $sy = (n + \delta)k$ to solve for the steady state capital/labor ratio,

$$k^{ss} = \left(\frac{sA}{n + \delta}\right)^{1/(1-\alpha)}$$

Now substitute this back into the per capita production function and solve for y

$$y = A^{1/(1-\alpha)} \left(\frac{s}{n + \delta}\right)^{\alpha/(1-\alpha)}$$

Finally, plug in the given numbers and solve for A

$$A = \left(\frac{30,000}{\sqrt{.2/.067}}\right)^{2/3} \approx 670.5$$

(b) When n drops to 0, the break-even investment line rotates down and you can see that the steady state capital/labor ratio will rise, and therefore, so will the steady state level of output per capita. In particular, the new state state per capita output will be

$$y^{ss} = (670.5)^{1.5}\sqrt{.2/.05} \approx \$34,724$$

The growth rate of total output will fall from 1.7% to 0. Finally, because the steady state capital/labor ratio will be higher, the steady state wage rate will rise.