# SIMON FRASER UNIVERSITY <br> Department of Economics 

## PROBLEM SET 2

(Due October 30)

1. (20 points). Consider the 2-period Fisherian model discussed in class. Suppose Yin and Yang have the same preferences. Also suppose Yin earns $\$ 100$ in period-1 and $\$ 100$ in period-2, while Yang earns nothing in period-1, but earns $\$ 210$ in period-2. Finally, suppose we observe that Yin and Yang both consume $\$ 100$ in each period. Given this information, what is the interest rate? Suppose interest rates increase. What happens to Yin's consumption? What happens to Yang's consumption? Are Yin and Yang better of worse off after the interest rate increase? Illustrate your answers with a graph.
2. (30 points). This question asks you to work through the complete (2-period) dynamic intertemporal model, for a particular specification of preferences and technology. Suppose the representative household's preferences are given by

$$
\begin{equation*}
U\left(C_{1}, C_{2}, \ell_{1}, \ell_{2}\right)=C_{1}+\gamma \ell_{1}^{1 / 3}+\beta\left\{C_{2}+\gamma \ell_{2}^{1 / 3}\right\} \tag{1}
\end{equation*}
$$

where $C_{1}$ and $C_{1}$ denote consumption in the first and second time period, $\ell_{1}$ and $\ell_{2}$ denote leisure in the first and second time period, $\gamma$ is a fixed parameter summarizing the relative preference for leisure, and $\beta<1$ is a fixed parameter summarizing the household's time preference. Output in each period is produced with the following Cobb-Douglas production function:

$$
\begin{equation*}
Y_{i}=z_{i} K_{i}^{2 / 3} N_{i}^{1 / 3} \quad i=1,2 \tag{2}
\end{equation*}
$$

where $z_{i}$ denotes total factor productivity in period- $i$. The economy begins with a fixed amount of capital, $K_{1}$, in period 1. This capital can be increased by investing in the first period, so that $K_{2}=K_{1}+I_{1}$. Notice for simplicity we've assumed that capital does not depreciate (i.e., $\delta=0$ ). As usual, the household confronts the following time constraint each period, $\ell_{i}+N_{i}=h$, where $h$ is the total time available in each period. Finally, for simplicity, suppose there is no government in this economy, and that all markets are perfectly competitive.

Calculate the competitive equilibrium values of consumption, employment and investment in each period. Also, derive expressions for the market clearing wage rates and interest rates. How do these variables depend on current and future productivity? [Hints: (i) Rather than look for market-clearing wage rates and interest rates, use the 'second welfare theorem', and compute the competitive equilibrium quantities by solving a 'social planner's problem'. That is, maximize the household's utility subject to the economy's technology and resource constraints. There are 5 constraints: $C_{i}+I_{i}=Y_{i}$, $\ell_{i}+N_{i}=h$ and $K_{2}=K_{1}+I_{1}$, where $Y_{i}$ is given by equation (2). That is, there are 2 aggregate resource constraints (i.e., the National Income Accouting identity), 2 time constraints, and a capital accumulation equation. (ii) Use the constraints to sub out ( $C_{1}, C_{2}, \ell_{1}, \ell_{2}$ ) and then solve an unconstrained maximization problem over $\left(N_{1}, N_{2}, I_{1}\right)$. (iii) Notice that since the economy ends in period 2, it makes no sense to invest in period 2. That is, we know $I_{2}=0$, so that $C_{2}=Y_{2}$.]

