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Department of Economics

Econ 305 Prof. Kasa
Intermediate Macroeconomic Theory Fall 2013

PROBLEM SET 2
(Solutions)

1. (20 points). Consider the 2-period Fisherian model discussed in class. Suppose Yin and Yang have the same preferences. Also suppose Yin earns \$100 in period-1 and \$100 in period-2, while Yang earns nothing in period-1, but earns \$210 in period-2. Finally, suppose we observe that Yin and Yang both consume \$100 in each period. Given this information, what is the interest rate? Suppose interest rates increase. What happens to Yin's consumption? What happens to Yang's consumption? Are Yin and Yang better of worse off after the interest rate increase? Illustrate your answers with a graph.

There are two ways to think about this. First, since Yin and Yang have the same preferences and make the same choices, their wealth must be the same. Alternatively, the present value of Yang's consumption must equal the present value of his income. Either way, we get the following equation,

$$100 + \frac{100}{1+r} = \frac{210}{1+r}$$

Solving for r we get $r = 10/100 = .10$. An increase in the interest rate produces both an income effect and a substitution effect. The substitution effect is always toward <u>less</u> current consumption. The income effect is positive for a lender, negative for a borrower, and zero for someone who, following Polonius' advice, is neither. Hence, Yin's consumption declines, since the income effect is zero. Yang's consumption also declines, since he is a borrower, and so the income effect reinforces the substitution effect. Yang is certainly worse off, while Yin is (locally) indifferent.

2. (30 points). This question asks you to work through the complete (2-period) dynamic intertemporal model, for a particular specification of preferences and technology. Suppose the representative household's preferences are given by

$$U(C_1, C_2, \ell_1, \ell_2) = C_1 + \gamma \ell_1^{1/3} + \beta\{C_2 + \gamma \ell_2^{1/3}\} \tag{1}$$

where C_1 and C_1 denote consumption in the first and second time period, ℓ_1 and ℓ_2 denote leisure in the first and second time period, γ is a fixed parameter summarizing the relative preference for leisure, and $\beta < 1$ is a fixed parameter summarizing the household's time preference. Output in each period is produced with the following Cobb-Douglas production function:

$$Y_i = z_i K_i^{2/3} N_i^{1/3} \qquad i = 1, 2 \tag{2}$$

where z_i denotes total factor productivity in period-i. The economy begins with a fixed amount of capital, K_1, in period 1. This capital can be increased by investing in the first period, so that $K_2 = K_1 + I_1$. Notice for simplicity we've assumed that capital does not depreciate (i.e., $\delta = 0$). As usual, the household confronts the following time constraint each period, $\ell_i + N_i = h$, where h is the total time available in each period. Finally, for simplicity, suppose there is no government in this economy, and that all markets are perfectly competitive.

Calculate the competitive equilibrium values of consumption, employment and investment in each period. Also, derive expressions for the market clearing wage rates and interest rates. How do these variables depend on current and future productivity? [Hints: (i) Rather than look for market-clearing wage rates and interest rates, use the 'second welfare theorem', and compute the competitive equilibrium quantities by solving a 'social planner's problem'. That is, maximize the household's utility subject to the economy's technology and resource constraints. There are 5 constraints: $C_i + I_i = Y_i$, $\ell_i + N_i = h$ and $K_2 = K_1 + I_1$, where Y_i is given by equation (2). That is, there are 2 aggregate resource constraints (i.e., the National Income Accouting identity), 2 time constraints, and a capital accumulation equation. (ii) Use the constraints to sub out (C_1, C_2, ℓ_1, ℓ_2) and then solve an unconstrained maximization problem over (N_1, N_2, I_1). (iii) Notice that since the economy ends in period 2, it makes no sense to invest in period 2. That is, we know $I_2 = 0$, so that $C_2 = Y_2$.]

Using the constraints to sub out C_1, C_2, ℓ_1 and ℓ_2 gives us the following unconstrained optimization problem:

$$\max_{N_1,N_2,I_1} \left[z_1 K_1^{2/3} N_1^{1/3} - I_1 + \gamma(h - N_1)^{1/3} + \beta\{z_2(K_1 + I_1)^{2/3} N_2^{1/3} + \gamma(h - N_2)^{1/3}\} \right]$$

The first-order conditions are:

$$\begin{aligned} N_1: &\quad \frac{1}{3} z_1 K_1^{2/3} N_1^{-2/3} - \frac{1}{3}\gamma(h - N_1)^{-2/3} &= 0 \\ N_2: &\quad \beta\{\frac{1}{3} z_2 (K_1 + I_1)^{2/3} N_2^{-2/3} - \frac{1}{3}\gamma(h - N_2)^{-2/3}\} &= 0 \\ I_1: &\quad -1 + \beta\frac{2}{3} z_2 (K_1 + I_1)^{-1/3} N_2^{1/3} &= 0 \end{aligned}$$

The first equation is just the usual $\frac{U_\ell}{U_c} = w$ condition for the first period, using the particular functional forms here, and using the fact that in a competitive equilibrium, $w = MPL$. The second equation is the same thing for the second period. The third equation is sometimes called an 'investment Euler equation'. It says that $U_{C_1} = (1+r)U_{C_2}$, where in a competitive equilibrium $(1+r) = MPK$. That is, an optimal intertemporal consumption/saving plan equates the marginal utility of today's consumption to the (discounted) marginal utility of tomorrow's consumption times the rate of return from investment.

The first equation can easily be solved for N_1 to get:

$$N_1 = \left(\frac{K_1}{K_1 + \left(\frac{\gamma}{z_1}\right)^{3/2}} \right) h$$

Note that $\gamma \uparrow \;\Rightarrow\; N_1 \downarrow$ and $z_1 \uparrow \;\Rightarrow\; N_1 \uparrow$, as you would expect. Substituting this expression into the equilibrium condition, $w = MPL = \frac{1}{3} z_1 K_1^{2/3} N_1^{-2/3}$, and then simplifying, gives us the following expression for the market-clearing wage rate in period 1,

$$w_1 = \frac{1}{3} \left(\frac{K_1 z_1^{3/2} + \gamma^{3/2}}{h} \right)^{2/3}$$

Note that first period wages rise when z_1 increases and when γ increases. In the first case, the labor demand curve shifts right. In the second case, the labor supply curve shifts left.

Solving for N_2 and I_1 is a bit more complicated because the two decisions are interrelated. That is, how much you invest in the first period depends on the marginal product of capital, which depends on how much you plan to work next period, since that influences the return from investment. Hence, we have *to solve two simultaneous equations. Still, it's not too bad. First, note that the first-order condition for I_1 implies*:

$$\left(\frac{K_1 + I_1}{N_2}\right)^{1/3} = \frac{2\beta z_2}{3} \tag{3}$$

while the first-order condition for N_2 implies:

$$z_2 \left(\frac{K_1 + I_1}{N_2}\right)^{2/3} = \gamma(h - N_2)^{-2/3} \tag{4}$$

Substituting equation (3) into equation (4) then gives us, $z_2 \left(\frac{2\beta z_2}{3}\right)^2 = \gamma(h - N_2)^{-2/3}$. *Solving for N_2 gives us the following expression for the equilibrium period 2 employment*:

$$N_2 = h - \gamma^{3/2} z_2^{-9/2} \left(\frac{2\beta}{3}\right)^{-3}$$

Note once again that employment increases with z, but decreases with γ. Finally, note that equation (3) implies $K_1 + I_1 = N_2 \left(\frac{2\beta z_2}{3}\right)^3$. *Substituting the equilibrium N_2 into this gives us the equilibrium first period investment*:

$$I_1 = \left(\frac{2\beta z_2}{3}\right)^3 h - \frac{\gamma^{3/2}}{z_2^{3/2}} - K_1$$

Note that first period investment increases when second period productivity increases. Less obviously, note that investment decreases when γ increases. A higher γ depresses equilibrium employment, which then depresses the marginal product of capital. Finally, to get the market-clearing interest rate, note that $1 + r = MPK = \frac{1}{2} z_2 (K_1 + I_1)^{-1/3} N_2^{1/3}$. *However, notice from equation (3) that this just implies* $(1 + r) = \frac{1}{\beta}$*!*.