SIMON FRASER UNIVERSITY
Department of Economics

Econ 305 — Prof. Kasa
Intermediate Macroeconomic Theory — Spring 2013

PROBLEM SET 2
(Solutions)

1. (20 points). Once again, the USA is currently worried about the supposedly disastrous effects of a looming 'fiscal cliff'. This so-called cliff involves automatic decreases in government spending. Use the dynamic intertemporal model developed in Chapter 11 to analyze the effects of this decrease in government spending. In particular, how would output, consumption, investment, and employment be affected? Illustrate your answers with graphs. How do the results depend on whether the cuts are permanent or transitory? Are people's concerns about the fiscal cliff justified? (Hint: Are the effects on output and employment the same as the effects on economic welfare?)

   *First, to be fully realistic, we should ask <u>why</u> government spending is being cut. Presumably, it's being cut because taxes were cut earlier, and now spending must be cut to keep the budget balanced, since Congress is not willing to raise taxes back up. From this perspective, the cuts could be permanent. This would produce a relatively strong wealth effect on labor supply and consumption demand. The positive wealth effect on consumption would (nearly) offset the decline in government spending, so there would be little or no effect on the $Y^d$ curve. At the same time, there would be a relatively strong wealth effect on labor supply. Households would <u>reduce</u> their labor supply, and this would shift the $Y^s$ curve left. As a result, since $Y^d$ is relatively constant, output would decline, and interest rates would rise. The higher interest rate would partially shift the $N^s$ back down, but not all the way. Wages would be higher, and employment would be lower.*

   *Of course, few changes in politics are permanent, so if households think the cuts are only temporary, then their increase in consumption would not offset the government decline, so the $Y^d$ would like shift down. At the same time, the wealth effect on labor supply would be weaker, so $Y^s$ would not shift out as much. Either way, market-clearing output and employment fall, but in the temporary case, you might see a decline in the interest rate.*

   *The main point here is that there is an important distinction between output and employment on the one hand, and economic welfare on the other hand. In Keynesian models, which exhibit market failures, the two typically move together, since low output is associated with underemployed resources. However, in the market-clearing model, falling output could actually be <u>good</u>, since low output is accompanied by less work and more leisure. There is no such thing as "involuntary unemployment" in the market-clearing model. So, although the model seems consistent with the dire predictions some people make about going over the 'fiscal cliff' (ie, output and employment will decrease), the welfare implications are quite different.*

2. (30 points). This question asks you to work through the complete (2-period) dynamic intertemporal model, for a particular specification of preferences and technology. Suppose the representative household's preferences are given by

$$U(C_1, C_2, \ell_1, \ell_2) = C_1 + \gamma\sqrt{\ell_1} + \beta\{C_2 + \gamma\sqrt{\ell_2}\} \tag{1}$$

where $C_1$ and $C_1$ denote consumption in the first and second time period, $\ell_1$ and $\ell_2$ denote leisure in the first and second time period, $\gamma$ is a fixed parameter summarizing the relative preference for leisure, and $\beta < 1$ is a fixed parameter summarizing the household's time preference. Output in each period is produced with the following Cobb-Douglas production function:

$$Y_i = z_i K_i^{1/2} N_i^{1/2} \qquad i = 1, 2 \tag{2}$$

where $z_i$ denotes total factor productivity in period-$i$. The economy begins with a fixed amount of capital, $K_1$, in period 1. This capital can be increased by investing in the first period, so that $K_2 = K_1 + I_1$. Notice for simplicity we've assumed that capital does not depreciate (i.e., $\delta = 0$). As usual, the household confronts the following time constraint each period, $\ell_i + N_i = h$, where $h$ is the total time available in each period. Finally, for simplicity, suppose there is no government in this economy, and that all markets are perfectly competitive.

Calculate the competitive equilibrium values of consumption, employment and investment in each period. Also, derive expressions for the market clearing wage rates and interest rates. How do these variables depend on current and future productivity? Here are some hints:

i Rather than look for market-clearing wage rates and interest rates, use the 'second welfare theorem', and compute the competitive equilibrium quantities by solving a 'social planner's problem'. (See chapter 5 in the textbook). That is, maximize the household's utility subject to the economy's technology and resource constraints. There are 5 constraints: $C_i + I_i = Y_i$, $\ell_i + N_i = h$ and $K_2 = K_1 + I_1$, where $Y_i$ is given by equation (2). That is, there are 2 aggregate resource constraints (i.e., the National Income Accouting identity), 2 time constraints, and a capital accumulation equation.

ii Use the constraints to sub out ($C_1$, $C_2$, $\ell_1$, $\ell_2$) and then solve an unconstrained maximization problem over $(N_1, N_2, I_1)$.

iii Notice that since the economy ends in period 2, it makes no sense to invest in period 2. That is, we know $I_2 = 0$, so that $C_2 = Y_2$.

iv To get the equilibrium wage rate and interest rate, substitute the equilibrium quantities into the appropriate optimality conditions.

*Using the constraints to sub out $C_1$, $C_2$, $\ell_1$ and $\ell_2$ gives us the following unconstrained optimization problem*:

$$\max_{N_1,N_2,I_1} \left[ z_1 K_1^{1/2} N_1^{1/2} - I_1 + \gamma\sqrt{h - N_1} + \beta\{z_2(K_1 + I_1)^{1/2} N_2^{1/2} + \gamma\sqrt{h - N_2}\}\right]$$

*The first-order conditions are*:

$$\begin{aligned}
N_1 : \qquad & \frac{1}{2} z_1 K_1^{1/2} N_1^{-1/2} - \frac{1}{2}\gamma(h - N_1)^{-1/2} &= 0 \\
N_2 : \qquad & \beta\{\frac{1}{2} z_2 (K_1 + I_1)^{1/2} N_2^{-1/2} - \frac{1}{2}\gamma(h - N_2)^{-1/2}\} &= 0 \\
I_1 : \qquad & -1 + \beta\frac{1}{2} z_2 (K_1 + I_1)^{-1/2} N_2^{1/2} &= 0
\end{aligned}$$

*The first equation is just the usual $\frac{U_\ell}{U_c} = w$ condition for the first period, using the particular functional forms here, and using the fact that in a competitive equilibrium, $w = MPL$. The second equation is the same thing for the second period. The third equation is sometimes called an 'investment Euler equation'. It says that $U_{C_1} = (1+r)U_{C_2}$, where in a competitive equilibrium $(1+r) = MPK$. That is, an* *optimal intertemporal consumption/saving plan equates the marginal utility of today's consumption to the (discounted) marginal utility of tomorrow's consumption times the rate of return from investment.*

*The first equation can easily be solved for* $N_1$ *to get*:

$$N_1 = \left( \frac{K_1}{K_1 + \left(\frac{\gamma}{z_1}\right)^2} \right) h$$

*Note that* $\gamma \uparrow \quad \Rightarrow \quad N_1 \downarrow$ *and* $z_1 \uparrow \quad \Rightarrow \quad N_1 \uparrow$*, as you would expect. Substituting this expression into the equilibrium condition,* $w = MPL = \frac{1}{2} z_1 K_1^{1/2} N_1^{-1/2}$*, and then simplifying, gives us the following expression for the market-clearing wage rate in period 1,*

$$w_1 = \frac{1}{2} \sqrt{\frac{K_1 z_1^2 + \gamma^2}{h}}$$

*Note that first period wages rise when* $z_1$ *increases and when* $\gamma$ *increases. In the first case, the labor demand curve shifts right. In the second case, the labor supply curve shifts left.*

*Solving for* $N_2$ *and* $I_1$ *is a bit more complicated because the two decisions are interrelated. That is, how much you invest in the first period depends on the marginal product of capital, which depends on how much you plan to work next period, since that influences the return from investment. Hence, we have to solve two simultaneous equations. Still, it's not too bad. First, note that the first-order condition for* $I_1$ *implies*:

$$\left( \frac{K_1 + I_1}{N_2} \right)^{1/2} = \frac{\beta z_2}{2} \tag{3}$$

*while the first-order condition for* $N_2$ *implies*:

$$z_2 \left( \frac{K_1 + I_1}{N_2} \right)^{1/2} = \gamma (h - N_2)^{-1/2} \tag{4}$$

*Substituting equation (3) into equation (4) then gives us,* $z_2 \left(\frac{\beta z_2}{2}\right) = \gamma(h - N_2)^{-1/2}$*. Solving for* $N_2$ *gives us the following expression for the equilibrium period 2 employment*:

$$N_2 = h - \frac{4\gamma^2}{\beta^2 z_2^4}$$

*Note once again that employment increases with* $z$*, but decreases with* $\gamma$*. Finally, note that equation (3) implies* $K_1 + I_1 = N_2 \left(\frac{\beta z_2}{2}\right)^2$*. Substituting the equilibrium* $N_2$ *into this gives us the equilibrium first period investment*:

$$I_1 = \left( \frac{\beta z_2}{2} \right)^2 h - \frac{\gamma^2}{z_2^2} - K_1$$

*Note that first period investment increases when second period productivity increases. Less obviously, note that investment decreases when* $\gamma$ *increases. A higher* $\gamma$ *depresses equilibrium employment, which then depresses the marginal product of capital. Finally, to get the market-clearing interest rate, note that* $1 + r = MPK = \frac{1}{2} z_2 (K_1 + I_1)^{-1/2} N_2^{1/2}$*. However, notice from equation (3) that this just implies* $(1 + r) = \frac{1}{\beta}$*!*.