SIMON FRASER UNIVERSITY

Department of Economics

Econ 305   Prof. Kasa

Intermediate Macroeconomic Theory   Spring 2013

PROBLEM SET 4

(Due April 12)

1. (20 points). Consider the Diamond-Dybvig model discussed in class and in Chapter 17. There are three periods: 0, 1, and 2. The economy consists of a large number, N, of consumers, each endowed with one unit of a good in period 0. This good can be used as an input in production, but the production process is 'illiquid' in the following sense: If one unit is invested in period 0 then $1+r$ units of the good are produced in period 2. If the production process is interrupted in period 1, then 1 unit is produced in period 1, and nothing more is produced in period 2. To be specific, assume that $r = 3$.

Consumers have random needs for cash. In particular, with probability t each consumer may need to consume in period 1. With probability $(1-t)$ a consumer is patient, and will only need to consume in period 2. Initially, in period 0, when investment decisions are made, consumers do not know which type they will be. They do not discover this until period 1. Hence, consumers maximize their expected utility

$$tU(C_1) + (1-t)U(C_2)$$

where C_1 denotes first-period consumption and C_2 denotes second-period consumption. To be specific, suppose that $t = 1/2$ and that the utility function takes the following form,

$$U(C) = a_0 - \frac{a_1}{C}$$

where the a_i parameters are positive constants.

(a) Suppose there is a competiive banking system. As outlined in class, compute the optimal contract (d_1, d_2).

(b) Explain why this economy is susceptible to bank runs. How large a fraction of people must be expected to withdraw in period-1 before a patient Type 2 person decides to run to the bank?

2. (20 points). Consider the following 'expectations-augmented Phillips Curve', discussed in Chpt. 18,

$$\pi - \pi^e = a(Y - Y^T)$$

where π is actual inflation, π^e is the public's expected inflation, Y is actual output, and Y^T is the trend (or 'natural') rate of output. Suppose the Central Bank's objective is to choose inflation so as to minimize the following social welfare function,

$$W(Y, \pi) = \alpha(\pi - \pi^*)^2 + \gamma(Y - Y^*)^2$$

where π^* is the inflation target, Y^* is the output target, and (α, γ) are fixed parameters summarizing the costs of inflation and output deviations.

(a) Calculate the Central Bank's optimal inflation rate as a function of the public's expected inflation rate. Suppose the public has 'Rational Expectations', so that $\pi^e = \pi$. What is the equilibrium inflation rate? What would the outcome be if the Central Bank could <u>commit</u> to an inflation rate ahead of time?

(b) Now suppose the above "game" is played repeatedly, and the Central Bank wants to minimize the expected present value of its losses,

$$E_0 \sum_{t=0}^{\infty} \beta^t W_t = E_0 \sum_{t=0}^{\infty} \beta^t [\alpha(\pi_t - \pi^*)^2 + \gamma(Y_t - Y^*)^2]$$

where $\beta < 1$ is the government's discount rate. As discussed in class, suppose the public starts out believing the government's announcements, but if the government ever 'cheats' by not following through on its promises, it will then lose faith in the government forever, and expect inflation to be at its one-period Nash equilibrium forever. Calculate how large β must be for the commitment outcome to be Nash equilibrium in this repeated game. (Hint: First calculate the one-period commitment payoff, the one-period 'cheating' payoff, and the the one-period Nash equilibrium payoff). Use the results to explain why Central Bankers usually have long-term appointments.