Econ 305   Prof. Kasa

Intermediate Macroeconomic Theory   Spring 2013

PROBLEM SET 4

(Due April 12)

1. (20 points). Consider the Diamond-Dybvig model discussed in class and in Chapter 17. There are three periods: 0, 1, and 2. The economy consists of a large number, N, of consumers, each endowed with one unit of a good in period 0. This good can be used as an input in production, but the production process is 'illiquid' in the following sense: If one unit is invested in period 0 then $1+r$ units of the good are produced in period 2. If the production process is interrupted in period 1, then 1 unit is produced in period 1, and nothing more is produced in period 2. To be specific, assume that $r = 3$.

Consumers have random needs for cash. In particular, with probability t each consumer may need to consume in period 1. With probability $(1 - t)$ a consumer is patient, and will only need to consume in period 2. Initially, in period 0, when investment decisions are made, consumers do not know which type they will be. They do not discover this until period 1. Hence, consumers maximize their <u>expected</u> utility

$$tU(C_1) + (1 - t)U(C_2)$$

where C_1 denotes first-period consumption and C_2 denotes second-period consumption. To be specific, suppose that $t = 1/2$ and that the utility function takes the following form,

$$U(C) = a_0 - \frac{a_1}{C}$$

where the a_i parameters are positive constants.

(a) Suppose there is a competiive banking system. As outlined in class, compute the optimal contract (d_1, d_2).

This is straight from the notes. Note that the given utility function is a version of the standard CRRA utility function, $U = \frac{C^{1-\rho}}{1-\rho}$, with $\rho = 2$. From the notes, we then know that

$$\frac{d_2}{d_1} = (1 + r)^{1/\rho} = \sqrt{4} = 2$$

Substituting into the budget constraint then yields

$$d_1 = \frac{\sqrt{1+r}}{1 - t + t\sqrt{1+r}} = \frac{4}{3}$$
$$d_2 = \frac{1+r}{1 - t + t\sqrt{1+r}} = \frac{8}{3}$$

Note that $d_1 > 1$ and $d_2 < (1 + r)$, so that the contract smooths consumption across states.

(b) Explain why this economy is susceptible to bank runs. How large a fraction of people must be expected to withdraw in period-1 before a patient Type 2 person decides to run to the bank?

Let f be the fraction of depositors who withdraw at date 1. Note that we might have $f > t$ if some Type 2 depositors show up. If fraction f show up at date 1, then fraction $f \cdot d_1$ of the investments must be liquidated to pay off depositors. This leaves

$$\hat{d}_2(f) = \frac{[1 - f \cdot d_1](1 + r)}{1 - f}$$

for each of the $1 - f$ depositors who stay. A Type 2 depositor will withdraw early if $\hat{d}_2(f) < d_1$, where $\hat{d}_2(f)$ represents his anticipated payoff based on an expectation of f period 1 withdrawals. Using the given parameter values, this then implies

$$f > \frac{4 - d_1}{r \cdot d_1} = \frac{2}{3}$$

2. (20 points). Consider the following 'expectations-augmented Phillips Curve', discussed in Chpt. 18,

$$\pi - \pi^e = a(Y - Y^T)$$

where π is actual inflation, π^e is the public's expected inflation, Y is actual output, and Y^T is the trend (or 'natural') rate of output. Suppose the Central Bank's objective is to choose inflation so as to minimize the following social welfare function,

$$W(Y, \pi) = \alpha(\pi - \pi^*)^2 + \gamma(Y - Y^*)^2$$

where π^* is the inflation target, Y^* is the output target, and (α, γ) are fixed parameters summarizing the costs of inflation and output deviations.

(a) Calculate the Central Bank's optimal inflation rate as a function of the public's expected inflation rate. Suppose the public has 'Rational Expectations', so that $\pi^e = \pi$. What is the equilibrium inflation rate? What would the outcome be if the Central Bank could <u>commit</u> to an inflation rate ahead of time?

Substituting the expectations-augmented Phillips curve into the welfare function yields

$$W = \alpha(\pi - \pi^*)^2 + \gamma \left[\frac{1}{a}(\pi - \pi^e) + (Y^T - Y^*)\right]^2$$

Differentiating with respect to π and equating to zero yields the following first-order condition

$$\left(\alpha + \frac{\gamma}{a^2}\right)\pi = \alpha\pi^* + \frac{\gamma}{a^2}\pi^e - \frac{\gamma}{a}(Y^T - Y^*) \tag{1}$$

Note that optimal inflation increases when <u>expected</u> inflation increases. Imposing the Rational Expectations condition, $\pi = \pi^e$, and solving for π, then yields the equilibrium inflation rate

$$\pi = \pi^* + \frac{\gamma}{a\alpha}(Y^* - Y^T)$$

Note that if the trend or 'natural' level of output is less than the target level of output, then equilibrium inflation will <u>exceeed</u> the target level of inflation. This is because the public knows the Central Bank has an incentive to engineer a surprise inflation (in order to increase output). As a result, they factor this into their expectations. Inflation rises up until the point that the Central Bank no longer has an incentive to increase inflation (because the marginal cost of inflation is now higher).

In equilibrium, $\pi = \pi^e = \pi^*$, and so $Y = Y^T$. Substituting these results into the welfare function the produces the following discretionary, or 'Nash', outcome

$$W_N = \gamma \left(1 + \frac{\gamma}{\alpha a^2}\right)(Y^* - Y^T)^2$$

Note that this outcome is suboptimal. Inflation is above its target, yet there is no benefit in terms of higher output. If the Central Bank could commit, it would promise to set $\pi = \pi^*$. Output would still equal its natural rate, which is below the target, but at least you would now avoid excessively high inflation. In terms of the welfare/loss function we would obtain the following commitment outcome

$$W_C = \gamma(Y^* - Y^T)^2$$

(b) Now suppose the above "game" is played repeatedly, and the Central Bank wants to minimize the expected present value of its losses,

$$E_0 \sum_{t=0}^{\infty} \beta^t W_t = E_0 \sum_{t=0}^{\infty} \beta^t [\alpha(\pi_t - \pi^*)^2 + \gamma(Y_t - Y^*)^2]$$

where $\beta < 1$ is the government's discount rate. As discussed in class, suppose the public starts out believing the government's announcements, but if the government ever 'cheats' by not following through on its promises, it will then lose faith in the government forever, and expect inflation to be at its one-period Nash equilibrium forever. Calculate how large β must be for the commitment outcome to be Nash equilibrium in this repeated game. (Hint: First calculate the one-period commitment payoff, the one-period 'cheating' payoff, and the the one-period Nash equilibrium payoff). Use the results to explain why Central Bankers usually have long-term appointments.

With only a one-period horizon, the commitment equilibrium is not credible. Why? Because if the public really believes that $\pi = \pi^*$ then the Central Bank has an incentive to set $\pi > \pi^e$. In particular, note that if we set $\pi^e = \pi^*$ in equation (1) above, we now get the following optimal inflation rate

$$\pi = \pi^* + \left[\frac{\gamma}{a(\alpha + \frac{\gamma}{a^2})} \right] (Y^* - Y^T) \tag{2}$$

Note that the Central Bank chooses to set inflation above its target. By doing so, it can increase output above the natural rate, and therefore bring output closer to its target. In particular, substituting $\pi^e = \pi^*$ and the optimal 'cheating' inflation rate in equation (2) into the Phillips Curve yields the following output level

$$Y = Y^T + \left[\frac{\gamma}{\gamma + \alpha a^2} \right] (Y^* - Y^T)$$

Substituting these into the welfare/loss function yields the following 'defect' loss

$$W_D = \gamma \left(\frac{\alpha a^2}{\gamma + \alpha a^2} \right) (Y^* - Y^T)^2$$

Note that $W_D < W_C$, which give the Central Bank an incentive to 'cheat' or 'defect', by engineering a surprise inflation. However, if this surprise inflation causes the public to stop trusting the Central Bank, this loss of reputation may be sufficient to deter the Central Bank from violating the public's trust. Specifically, if the following condition holds, then the one-time benefit will be less than the present value of future losses, and reputation can sustain the commitment outcome

$$(W_c - W_D) < \frac{\beta}{1 - \beta}(W_N - W_C)$$

The left-hand side is the one-time benefit, while the right-hand side is the present value of future losses. Substituting in our previous results, we get

$$\frac{\alpha a^2}{\gamma + \alpha a^2} < \frac{\beta}{1 - \beta}$$

Note the $(Y^* - Y^T)^2$ terms cancel out.