

SIMON FRASER UNIVERSITY  
Department of Economics

Econ 808  
Macroeconomic Theory

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FINAL EXAM - December 5

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (10 points each).

1. Empirical evidence suggests that per capita incomes are converging across countries.
2. Competitive equilibria in Overlapping Generations models are dynamically inefficient when equilibrium interest rates exceed the economy's growth rate.
3. Consumption taxes discourage investment and capital accumulation.
4. The equity premium puzzle can be explained with sufficiently volatile idiosyncratic risk.

The following questions are short answer. Be sure to explain and interpret your answer.

5. (30 points). **Endogenous labor in the Cass-Koopmans Model.** In class, we assumed labor was supplied inelastically in the Cass-Koopmans model. This question extends the model to consider endogenous labor supply, and uses it to discuss the effects of fiscal policy.

Consider a competitive economy with a representative household that maximizes the following utility function

$$U_0 = \max_{C(t), L(t)} \int_0^{\infty} [\gamma \log C(t) + (1 - \gamma) \log(1 - L(t))] e^{-\rho t} dt$$

subject to the following flow budget constraint,

$$\dot{A}(t) = r(t)A(t) + w(t)L(t) - C(t) - T(t)$$

where  $L(t)$  denotes labor supply,  $w(t)$  is the wage rate,  $A(t)$  is the household's asset holdings,  $r(t)$  is the interest rate, and  $T(t)$  is a lump-sum tax.

Assume competitive firms produce a single good using inputs of labor and capital with the following production function

$$Y(t) = K(t)^\alpha L(t)^{1-\alpha}$$

and assume that capital depreciates at the instantaneous rate  $\delta$ .

Finally, assume the government uses its lump-sum tax revenue,  $T(t)$ , in order to finance government purchases  $G(t)$ , and that these purchases provide no utility to the household.

- (a) Write down the current value Hamiltonian that characterizes the household's dynamic optimization problem. Derive the first-order conditions (including the relevant transversality condition).
- (b) Use part (a) and the second welfare theorem to characterize a competitive equilibrium. That is, derive a dynamic system that describes how  $C(t)$ ,  $K(t)$ , and  $L(t)$  evolve over time. (Hints: (1) Impose the equilibrium condition  $A(t) = K(t)$ , (2) Show that  $L(t)$  can be written as a function of  $C(t)$  and  $K(t)$ , and use it to reduce the system to a standard two-dimensional system in  $C(t)$  and  $K(t)$ ).
- (c) Illustrate the economy's dynamics with a phase diagram, with  $C$  on the vertical axis and  $K$  on the horizontal. Be sure to include the saddlepath. How does this picture compare to the one in class, where labor was exogenous?

(d) Consider an initial steady state, where  $G(t) = \bar{G}_0$  is constant. Beginning from this steady state, suppose  $G$  increases unexpectedly to  $\bar{G}_1 > \bar{G}_0$ , and is expected to remain at this higher level permanently. (Does it matter whether this increased spending is financed by current taxes or debt?). Use the above phase diagram to illustrate the economy's dynamic adjustment to this fiscal shock. What happens to investment and long-run output? Explain intuitively. Does this conclusion support recent claims by US Republicans that increased government spending crowds out private investment and reduces output?

6. (30 points). **Market Liberalization and Interest Rate Volatility.** Consider a world with two countries, indexed by  $i = a, b$ . Each country is inhabited by a representative household with the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_{1t}^\alpha C_{2t}^{1-\alpha})^{1-\gamma}}{1-\gamma}$$

where  $C_1$  and  $C_2$  denote consumption of two distinct types of (nondurable) goods. The representative household in country- $i$  owns one tree that yields *both* types of goods in each period - a random amount  $e_{1t}^i$  of good 1, and a constant amount of 1 for good 2. Each period  $e_{1t}^a$  takes on one of two values:  $e_h > 1/2$  with probability  $p$ , or  $1 - e_h$  with probability  $1 - p$ . Finally, suppose that the total world endowment of good 1 is always constant at  $e_{1t}^a + e_{1t}^b = 1 \forall t$ . (Hence, endowments of good 1 are perfectly negatively correlated across countries).

Assume that good 2 is the numeraire, so that its price is always 1. Suppose that in addition to the goods market, there is a market for claims to trees (which yield both types of consumption goods), and a market for riskless one-period bonds. (Remember that a riskless bond has a date- $t$  price of  $1/(1 + r_{f,t})$ , where  $r_{f,t}$  is the riskless interest rate, and a date- $(t + 1)$  payoff of 1 unit of the numeraire (good 2).

- (a) Suppose first that countries live in autarky, meaning that they trade neither goods nor assets. Let  $b_t$  denote the household's holdings of bonds, and  $s_t$  be its holdings of equity claims to trees. Using this notation, define a competitive equilibrium. Write down the household's budget constraint. Describe the market-clearing conditions. (Note: You only need to do this for one country).
- (b) Write down the household's Bellman equation, and derive the first-order (and envelope) conditions. Use them to derive the asset-pricing Euler equations. Use these to derive expressions for equilibrium stock and bond prices. (Hints: (1) Impose market-clearing in the household's Euler equations; (2) Interest rates should be state-dependent, i.e., depend on the current realization of  $e_{1t}^i$ ; (3) As before, you only need to do this for one of the countries).
- (c) Now suppose there is free trade in both goods and assets each period. Derive new expressions for the riskless interest rate in each country. Compare to your results in part (b).
- (d) Now suppose there is free trade in *goods*, but financial markets are *closed* (ie., there is no trade in assets). Derive expressions for the equilibrium goods prices and the riskless interest rates in each country.
- (e) Finally, assume  $\alpha = 1/2$ ,  $\gamma = 3$ , and  $e_h = 3/4$ , and derive explicit formulas for the riskless interest rate under the three alternative market structures (ie., autarky, free trade in goods and assets, and free trade in goods only). Given these results, comment on the effects of market liberalization on interest rate volatility. Explain intuitively.