SIMON FRASER UNIVERSITY
Department of Economics

Econ 808 | Prof. Kasa
Macroeconomic Theory | Fall 2020

FINAL EXAM - December 3
(Due December 7, 6pm)

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (5 points each).

1. If markets are efficient, then stock prices follow random walks.
2. The convergence rate is faster in the Cass-Koopmans model than in the Solow model.
3. In the Lucas model, good news about future dividends increases current stock prices.
4. Empirical evidence suggests that per capita incomes are converging across countries.

   The following questions are short answer. Be sure to explain and interpret your answer.

5. (20 points). **Rare Disasters and the Equity Premium**. Consider an economy with just two states: "normal" and "disaster". There is a single risky asset in the economy that yields a nonstorable 'dividend', which represents the only consumption good in the economy. In the normal state dividend (and consumption) growth is 3.0%, and in the disaster state it is -20%. Suppose agents have identical constant relative risk aversion preferences, with coefficient of relative risk aversion of 2.0, and a time discount factor of $\beta = .99$. Finally, assume that the state is i.i.d. over time, and that the probability of the normal state is .90 and the probability of the disaster state is .10.

   (a) What is the implied risk-free rate in this economy? (Hint: Use the agent's Euler equation).

   (b) Suppose the rate of return on the risky asset is 17% in the normal state. What must be the rate of return in the disaster state? What is the implied equity premium?

6. (20 points). **Nonrenewable Resources and Growth**. Consider a standard Solow-style growth model with *three* inputs: Capital (K), Labor (L), and Energy (E). The flow of output is given by

$$Y(t) = A(t)K(t)^{\alpha}E(t)^{\gamma}L(t)^{1-\alpha-\gamma}$$

   where $g_A = \dot{A}/A$ is the exogenous rate of technological progress, and $n = \dot{L}/L$ is the exogenous rate of population (labor force) growth. Suppose there is a fixed stock of nonrenewable resources, R, and that energy use depletes this stock, so that

$$\dot{R} = -E$$

   For simplicity, suppose that a constant fraction, $0 < \eta < 1$, of the remaining stock is used at each point of time, so that $E = \eta R$.

   (a) Letting R_0 denote the initial stock of resources, derive an expression for the remaining stock at each moment of time. (Hint: Solve a simple linear differential equation). Use this to derive an expression for energy use at each moment of time.

   (b) For simplicity, suppose that the saving rate is constant, so that $\dot{K} = sY - \delta K$. Prove that the economy converges to a balanced growth path, and that K/Y is constant along this path. Using eq. (1) and the fact that K/Y is constant, calculate the growth rate of per capita output on the balanced growth path as a function of g_A, η, and n.

(c) Suppose that initially the economy uses energy at the rate $\eta = .04$, so that the half-life of the resource stock is $\ln(2)/.04 \approx 17$ years. Also suppose that capital share's of GDP is 20% ($\alpha = .20$), and that energy's share is 10% ($\gamma = .10$). Finally, suppose that as part of an international agreement the country must permanently cut its energy use to $\eta = .01$ (so now the half-life of the remaining stock is 69 years). Calculate the effects of this policy change on output. Be sure to distinguish between the immediate effect, and the effect on the long-run growth rate. Is there a trade-off between the short-run and long-run effects of this policy?

7. (20 points). **Public Infrastructure and Endogenous Growth**. Consider a standard continuous-time Cass-Koopmans economy. A representative household has preferences

$$U = \max_{c} \int_0^\infty \frac{C(t)^{1-\gamma}}{1-\gamma} e^{-\rho t} dt$$

The household earns income by supplying labor (inelastically) and renting capital to a competitive firms. Factor markets are competitive. The twist here is that the production function now incorporates a stock of public (nonrival) infrastructure goods, e.g., roads and bridges. (In reality, roads and bridges are not nonrival, especially in Vancouver! But ignore that). These publicly provided goods increase the productivity of labor and capital, but are taken as given by the competitive firms. Specifically, the production function now takes the form (note, for simplicity, L is assumed constant):

$$Y(t) = AG(t)^\phi K(t)^\alpha L^{1-\alpha}$$

where $G(t)$ represents the current stock of public infrastructure, and ϕ is a constant parameter, which captures the productivity of public infrastructure. Finally, suppose that $G(t)$ is financed by lump-sum taxes. (Does it matter whether the government balances its budget each period?)

(a) Let $r(t)$ be the market interest rate. Write down the household's Euler equation describing its optimal consumption/saving plan.

(b) Now suppose $\phi = 1 - \alpha$. Assume the government chooses G so as to maintain a constant G/Y ratio (which is not a bad assumption empirically). Derive an expression for the marginal product of capital as a function of G/Y. Using the fact that $MPK = r + \delta$ and your answer to part (a), state conditions under which this economy exhibits endogenous growth. What is the equilibrium growth rate? What would happen if $\phi < 1 - \alpha$? Explain.

(c) Let's now consider the *optimal* choice of G. Continue to assume that $\phi = 1 - \alpha$. Consider a social planner that simultaneously selects paths of $C(t)$ and $G(t)$ so as to maximize the present discounted value of the household's utility, subject to the economy-wide resourced constraint

$$\dot{K} = Y - C - \delta K - G$$

Write down the planner's Hamiltonian (either current or present-value, your choice) and derive the first-order optimality conditions. Prove that the optimal path of $G(t)/Y(t)$ is constant, and derive an expression for the optimal G/Y ratio. How does it compare to observed data?

8. (20 points). **New Technologies and the Stock Market**. Consider an economy comprised of a large number of households, each with a utility function given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \qquad 0 < \beta < 1$$

where u is twice differentiable, increasing, and strictly concave. (If necessary, assume it satisfies the Inada conditions). Initially, there is only one technology. Label it technology 1. This technology produces a flow of (nondurable) output according to

$$y_{1t} = z_1 F(a_{1t}, n_{1t})$$

where a_{1t} is land used at time-t and n_{1t} is labor input at time-t. Assume F is twice differentiable, homogeneous of degree one, concave, and features strictly decreasing marginal products of both factors. Suppose that the marginal product of labor satisfies the Inada condition,

$$\lim_{n \to 0} z_1 F_n(a, n) = \infty$$

Suppose that land and labor are in fixed supply, and normalize their aggregate quantities to one. Suppose agents are free to trade all the assets they want (e.g., bonds of all maturities and ownership shares in firms). Note that with constant returns to scale we can think of there being a single firm. Assume the firm owns all the land, so that firm shares represent claims to the flow of land rents.

Suppose that at time 0 there is an announcement that at time $T > 0$ a new technology will become available. This technology will be able to produce the same good according to the production function

$$y_{2t} = z_2 n_{2t}$$

where z_{2t} is a bounded stochastic process that satisfies the property

$$z_{2t} > z_1 F_n(1,1) \qquad \forall t$$

and where n_{2t} is labor employed in the new industry. Note that the announcement only specifies the distribution of z_{2t} for $t \geq T$, and not the actual realizations. (The actual values of z_{2t} do not become known until time-t.

(a) Calculate the equilibrium level of output, land rental rate, wage rate, and interest rate before the announcement. Given the land rental rate, what is the stock market value of the firm?

(b) Describe the effect of the announcement on both short-term bonds (i.e., with maturities less than T) and long-term bonds (with maturities greater than T).

(c) Describe the effect of the announcement on expected wages, $E_0 w_t$ for $t = 0$ and $t = T + k, k > 0$. (Workers are free to work in whichever industry pays the higher wage).

(d) Describe the response of the stock market to the announcement (i.e., what happens to the aggregate price of land?).

(e) Does the answer to part (d) sound a note of caution about using the stock market as a signal of the future health of the economy?

9. (100 points). **Bonus Question**: Prove that every even integer greater than 2 is the sum of two prime numbers.