SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 | Prof. Kasa
Financial Economics, I | Fall 2023

FINAL EXAM
Due December 12, 6pm

Answer the following True, False, or Uncertain. Explain Why. 10 points each.

1. Good news about future dividends increases stock prices.
2. If markets are efficient, then stock prices follow random walks.
3. According to Grossman-Stiglitz (1980), when the variance of noise/liquidity trading increases, asset prices become less informative.
4. Governments should try to prevent asset market bubbles from occurring.

Short answer questions:

5. (35 points). **Rare Disasters and the Equity Premium**. A dataset (excel spreadsheet) named Shiller-Ch26-data is posted on the course webpage. It contains annual data from 1871-2016 on stock returns, interest rates, and per capita consumption for the US economy. The only 3 columns you need are column H (the 1-year real interest rate), column I (real per capita consumption), and column P (real return on the S& P500). Note, since the consumption data only run from 1889-2009, in what follows just use this shorter sample when plotting and computing moments.

   (a) Use the consumption data to compute a series of consumption growth rates, C_{t+1}/C_t. Also, since the stock return data is in percentage units, add 1.0 to it to express in units of gross returns. Now display time-series plots of the interest rate, stock return, and consumption growth series. Calculate their means. What is the equity premium?

   (b) Using the procedure outlined in Lecture Slide 14 (pages 15-16) compute and plot the Hansen-Jagannathan bound (put μ_m on the horizontal axis and σ_m on the vertical axis. Also, compute the mean and standard deviation of the implied stochastic discount factor when agents have time-additive, CRRA preferences, with $\beta = .99$ and $\gamma = 1, 2, \cdots 40$. (Hints: Use the data on consumption growth and let the computer do the work!) Plot the theoretical stochastic discount factors along with the Hansen-Jagannathan bound. Your plot should resemble the one reported on page 17 of Lecture 14. Explain intuitively why increases in γ first cause the mean SDF to decrease, and then eventually causes it to increase.

   (c) Consider an economy governed by a 2-state Markov chain. Let State 1 denote 'normal times' and let State 2 denote a 'recession'. Let G_i denote (gross) consumption growth in state-i and R_i denote stock returns in state-i. Using this notation, we can write the investor's Euler equation as

$$1 = \beta \sum_{j=1}^{2} \pi_{ij} (G_j)^{-\gamma} R_j \qquad i = 1, 2$$

   where π_{ij} denotes the probability of going from state-i to state-j. Assume that $\beta = .99$ and $\gamma = 5$. Also, assume that in recessions, consumption declines by 3% (ie, $G_2 = .97$) and that the probability of switching from expansion to recession is 8% (ie, $\pi_{12} = .08$) Next, calibrate the value of the π_{21} so that recessions occur every 10 years on average (Hint: the long-run probability of being in state 1 is $\Pi_1 = \pi_{21}/(\pi_{12} + \pi_{21})$ and the long-run probability of being in state 2 is $\Pi_2 = 1 - \Pi_1$.) Finally, calibrate growth during normal times, G_1, so that the model's mean consumption growth matches the data. (Hint: mean consumption growth is $\Pi_1 G_1 + \Pi_2 G_2$).

Given all these parameter values, use the (state-dependent) Euler equations to calculate state-dependent values for the risk-free rate and stock return. (Hint: you need to solve 2 simultaneous linear equations for the stock return). What is the mean equity premium?

(d) Now consider a 3-state economy, featuring a small probability of a disaster. Assume that when disasters occur, consumption declines by 30%. Also, assume that disasters only last 1-period, after which you return to normal growth (ie, $\pi_{31} = 1$), and that disasters only occur while in the normal state (ie, $\pi_{23} = 0$). Finally, assume that the probability of switching to a disaster is $\pi_{13} = .02$ and subtract .01 from each of the previously calculated values of π_{11} and π_{12}, so that probabilities continue to sum to 1. Now repeat the analysis in part (c) to calculate the mean equity premium. (Hints: (1) the long-run stationary distribution of a Markov chain is given by the eigenvector associated with the unit eigenvalue of the transition matrix. (2) Now you need to solve 3 simultaneous linear equations to get the stock return in each state).

(Hint: This question is based on a 2006 paper by Barro entitled "Rare Disasters and Asset Markets in the Twentieth Century" in the *Quarterly Journal of Economics*).

6. (25 points). **Heterogeneous Beliefs and Bubbles**. Consider an economy that fluctuates between two states – "Good Times/Bad Times". Suppose an asset yields a (nonstorable) consumable dividend of 1 during G times, and 0 during B times. The economy is inhabited by 2 types of risk-neutral agents - G-optimists, who think that G times will last a relatively long time, and B optimists, who think B times will be relatively short-lived. In particular, letting state-1 denote B times and state-2 denote G times, the beliefs of G-optimists and B-optimists are summarized by the following two Markov transistion matrices:

$$Q^g = \begin{pmatrix} 1/2 & 1/2 \\ 1-\pi_g & \pi_g \end{pmatrix} \qquad Q^b = \begin{pmatrix} \pi_b & 1-\pi_b \\ 1/2 & 1/2 \end{pmatrix}$$

where by assumption $\pi_g > 1/2 > \pi_b$. Following Harrison and Kreps (1978), suppose agents do not update their beliefs and that no short-sales are allowed.

(a) Suppose the asset trades 'ex-dividend', so that dividends are not received (if at all) until after you buy the asset. Suppose agents have the same rate of time preference, $\beta = 3/4$. Also suppose $\pi_g = 3/4$ and $\pi_b = 1/3$. Calculate each agent's the buy-and-hold valuation for the asset. If no trading is allowed, who buys the asset? What are the prices in each state?

(b) Now suppose trading is allowed. What is the equilibrium price? Describe the trading dynamics. Following Scheinkman and Xiong (2003), what is the 'bubble' component of the price? How does it vary across states?

(c) Bonus question for Ph.D students: Suppose the actual (unknown) transition probabilities are $(\bar{\pi}_b, 1 - \bar{\pi}_b)$. Prove that the agent whose beliefs are closest to the truth (in the sense of relative entropy) will eventually accumulate all the wealth in the economy. (See Appendix 8B in Ljungqvist & Sargent, 4th ed.).

(Hint 1: This question is based on Lecture 16 (pgs. 8-10). Hint 2: In principle, you could do the matrix inversions and multiplications 'by hand', since there are just 2 states. However, I strongly recommend that you let your computer do the work.)