The Backus-Smith (JIE, 1993) Puzzle

- The observed weak correlation of consumption across countries is a puzzle from the perspective of standard risk-sharing models.

- By themselves, (realistic) shipping costs (in a 1-good model) don't really help. (BKK result).

- Introducing incomplete financial markets can help if productivity shocks are idiosyncratic and very persistent (Baxter + Crucini), or if countries lack commitment (Kehoe + Perri).

- Today we examine an even simpler solution. Suppose there are multiple goods, and some goods are non-traded.

- We will see that NT goods can resolve the consumption correlation puzzle, but they give rise to another puzzle, i.e., the 'Backus-Smith Puzzle', which relates to the correlation between relative consumption growth + changes in real ex. rates.

Assumptions

1.) There are I countries. Each country is endowed with w_i units of a single (identical) traded good, and x_i units of a country-specific NT good.

2.) Consumption of the T good in country i is denoted by a_i, and consumption of its NT good is denoted by b_i. Preferences are identical across countries, and given by:

$$\boxed{u(a,b) = \frac{\left\{\left[\alpha a^{\rho} + (1-\alpha) b^{\rho}\right]^{1/\rho}\right\}^{1-\gamma}}{1-\gamma}}$$

$1/\gamma$ = Intertemporal elasticity of substitution

$\frac{1}{1-\rho}$ = Intratemporal elasticity of substitution between a and b.

3.) Financial markets are complete. This means that the competitive equilibrium solves a Pareto planning problem, subject to the constraint that some goods are non-traded.

Quantity & Prices Indices

The relevant quantity index is given by:

$$\boxed{c(a,b) = \left[\alpha a^{\rho} + (1-\alpha) b^{\rho}\right]^{1/\rho}}$$

And the consumption-based price index is:

$$\boxed{P(q_0, q_i) = \left[\alpha^{\frac{1}{1-\rho}} q_0^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} q_i^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}}$$

where q_0 is the (common) price of the T good, and q_i is the (country-specific) price of NT goods.

Real Exchange Rates

Define the real ex. rate between countries i and j as follows:

$$\boxed{e_{ij} = \frac{P_j(q_0, q_j)}{P_i(q_0, q_i)}}$$

with this definition $e_{ij} \uparrow$ implies a real depreciation of country i's currency

Planner's Problem

$$\max_{a_i, b_i} \sum_{i=1}^{I} \lambda_i U_i = \sum_{i=1}^{I} \lambda_i \sum_{t=0}^{T} \beta^t \sum_{z^t} \pi(z^t) u[a_i(z^t), b_i(z^t)]$$

subject to

1.) $\sum_{i=1}^{I} a_i(z^t) \leq W(z^t) = \sum_{i=1}^{I} w_i(z^t) \quad \forall z^t$

2.) $b_i(z^t) \leq x_i(z^t) \quad \forall i, z^t$

FOCs

Exploiting the connection between Lagrange Multipliers and prices, denote the LM on the T good constraint by $\beta^t \pi(z^t) q_0(z^t)$ and denote the LM on the NT constraints by $\beta^t \pi(z^t) q_i(z^t)$. We can the write the FOCs as follows:

$$\boxed{\begin{aligned} \lambda_i \frac{\partial u(\cdot)}{\partial a_i} &= q_0 \\ \lambda_i \frac{\partial u(\cdot)}{\partial b_i} &= q_i \end{aligned}}$$

Using the above utility function we can then write the FOCs as follows:

$$q_0 = \lambda_i c_i^{1-\rho-\gamma} \alpha a_i^{\rho-1}$$

$$q_i = \lambda_i c_i^{1-\rho-\gamma} (1-\alpha) b_i^{\rho-1}$$

Multiply the first by a_i and the second by b_i, and then add:

$$\begin{aligned} a_i q_0 + b_i q_i &= p_i c_i \\ &= \lambda_i c_i^{1-\rho-\gamma} \left[a_i^{\rho} \alpha + (1-\alpha) b_i^{\rho} \right] \end{aligned}$$

Multiply both sides by c_i^{-1}

$$p_i = \lambda_i c_i^{-\gamma}$$

Finally, take ratios

$$\boxed{\frac{p_j}{p_i} = e_{ij} = \left(\frac{\lambda_j}{\lambda_i}\right)\left(\frac{c_i}{c_j}\right)^{\gamma}}$$

Implications

Note that with complete mkts. the λ_i's are constant (contrast with Kehoe/Perri). Taking log difference therefore implies:

$$\boxed{\Delta \log\left(c_i/c_j\right) = 1/\gamma \, \Delta \log\left(e_{ij}\right)}$$

$\Rightarrow$ Countries with (relatively) rapidly growing consumption should have depreciating real exchange rates.

3 Testable Predictions

1.) $\text{mean}\left[\Delta \log\left(c_i/c_j\right)\right] = 1/\gamma \ \text{mean}\left[\Delta \log\left(e_{ij}\right)\right]$

2.) $\text{st. dev.}\left[\Delta \log\left(c_i/c_j\right)\right] = 1/\gamma \ \text{st. dev}\left[\Delta \log\left(e_{ij}\right)\right]$

3.) $\text{autocorr}\left[\Delta \log\left(c_i/c_j\right)\right] = \text{autocorr}\left[\Delta \log\left(e_{ij}\right)\right]$

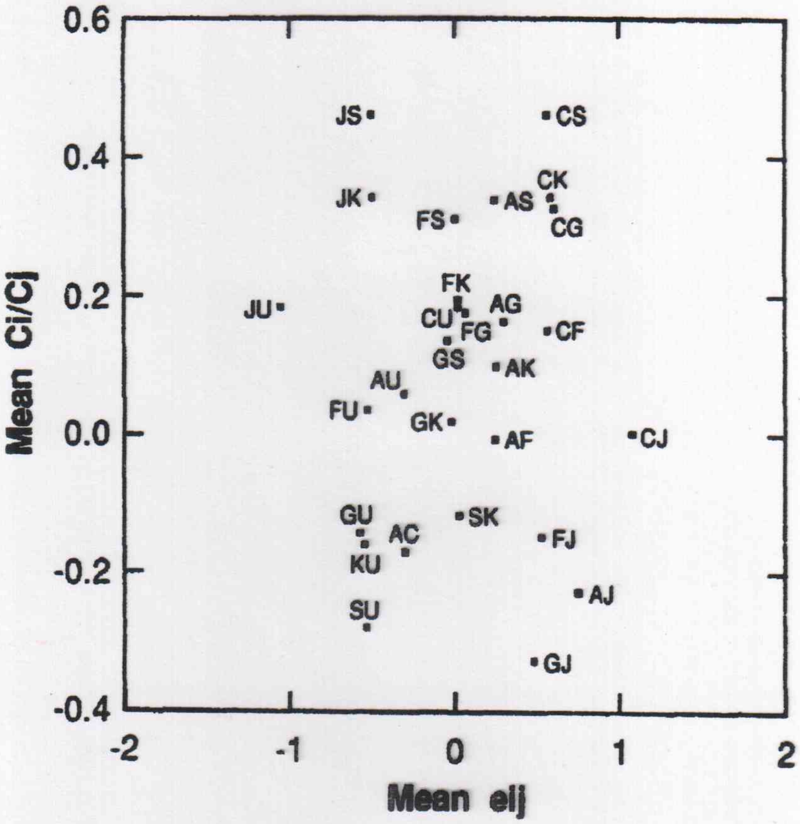

Fig. 3. Means (growth rates).

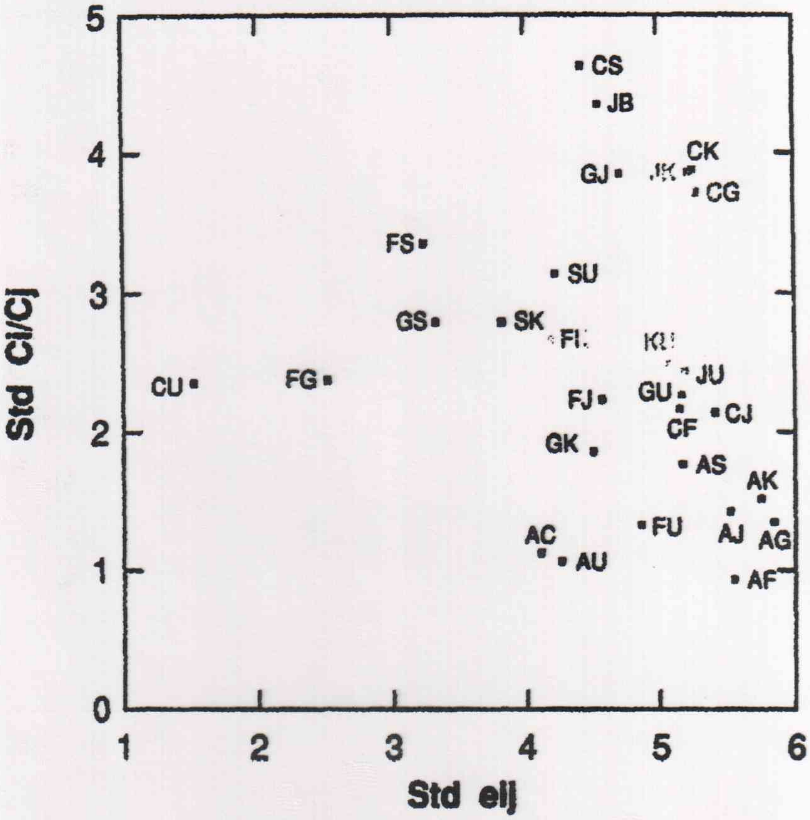

Fig. 1. Standard deviations (growth rates).

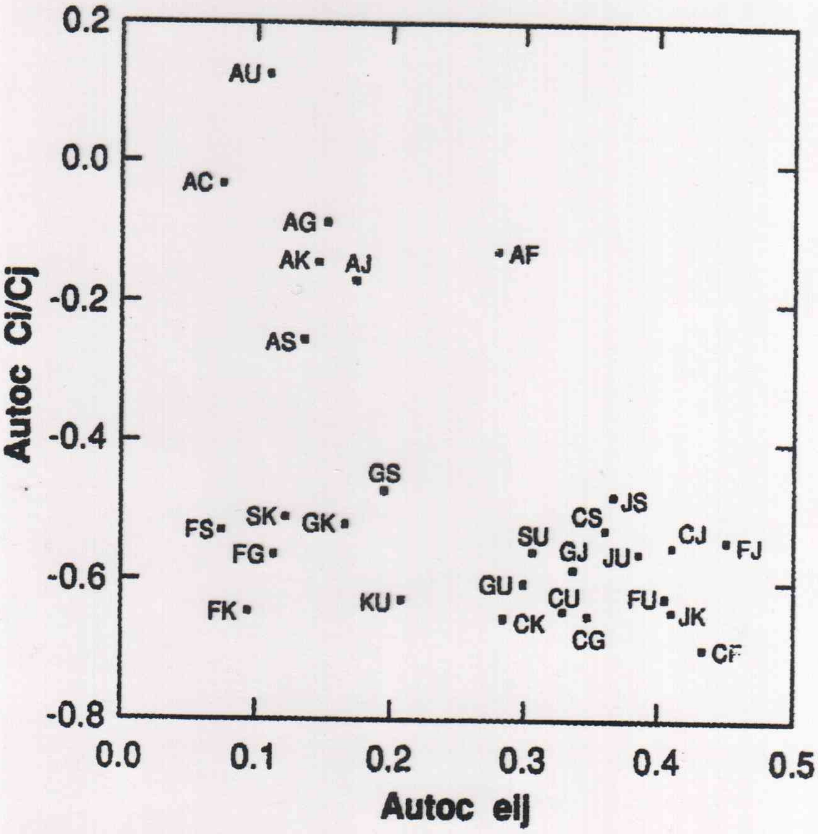

Fig. 2. Autocorrelations (growth rates).

Potential Resolutions of the Backus-Smith Puzzle

1.) Preference/Demand Shocks

- with only supply/endowment shocks

$c^i_{NT} \uparrow \Rightarrow q_i \downarrow \Rightarrow P_i \downarrow \Rightarrow e_{ij} \uparrow$

Demand shocks could offset this

2.) Non-separable preferences (leisure).

- Efficiency equates MU, not consumption per se.

3.) Incomplete mkts. [Chari, Kehoe, McGrattan (2002)]

- Now have

$$E_t\left\{\frac{u'(c_{t+1})/P_{t+1}}{u'(c_t)/P_t}\right\} = E_t\left\{\frac{u'(c^*_{t+1})/P^*_{t+1}}{u'(c^*_t)/P^*_t}\right\}$$

- with diff. productivity, a Balassa-Samuelson effect could dominate the adverse TOT effect.