

# Topics

1.) Intl. Capital Mkts. with Lack of Commitment

- Sovereign Risk + Default
- One-Sided Commitment (Small Country Case)

2.) Caveats

- Saving
- Bulow-Rogoff (1989)

3.) 2-sided Lack of Commitment

- Last time we saw that non-traded goods and dynamic spanning with bond trading could provide partial resolutions of the portfolio home bias puzzle
- However, the most important explanation likely stems from capital market imperfections.
- There are two main sources of imperfection:  
(1) Lack of commitment, and (2) Asymmetric Info.  
Today we explore the consequences of lack of commitment.
- So far, we have implicitly assumed an infinite penalty for default. This is not realistic, especially for sovereign lending!

### Assumptions

- 1.) A small risk averse country faces risky endowment.  
The country can't commit to contracts.
- 2.) There are competitive risk neutral foreign lenders who can commit to deliver resources.
- 3.) If the country defaults it is shut out of the intl. capital market forever. (lives in autarky).

## Preferences and Constraints

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_s)$$

$$c_s = \bar{Y} + \varepsilon_s - P_s(\varepsilon_s)$$

$$\begin{aligned}\varepsilon_s &\sim i.i.d. E[\underline{\varepsilon}, \bar{\varepsilon}] \\ E(\varepsilon_s) &= 0\end{aligned}$$

$P_s(\varepsilon_s)$  = insurance payment

## Zero Profit Condition

$$\sum_{i=1}^N \pi(\varepsilon_i) P(\varepsilon_i) = 0 \quad \text{Note: } i.i.d. \Rightarrow P_i = P$$

(Note: Period-by-period contracts)

## Arrow-Debreu Contract

$$P(\varepsilon_s) = \varepsilon_s \implies c_s = \bar{Y} \quad \forall s \text{ (full insurance)}$$

Now consider default,

$$\text{Gain}(\varepsilon_+) = u(\bar{Y} + \varepsilon_+) - u(\bar{Y})$$

$$\begin{aligned}\text{Cost} &= \sum_{s=t+1}^{\infty} \beta^{s-t} u(\bar{Y}) - \sum_{s=t+1}^{\infty} \beta^{s-t} E_s u(\bar{Y} + \varepsilon_s) \\ &= \frac{\beta}{1-\beta} [u(\bar{Y}) - E u(\bar{Y} + \varepsilon)] \quad \left. \right\} \text{by stationarity} \\ &\approx \frac{\beta}{1-\beta} \frac{1}{2} |u''(\bar{Y})| \text{var}(\varepsilon)\end{aligned}$$

Note, gain from default is greatest at  $\bar{\varepsilon}$ .  
Sustainability requires  $\text{Cost} > \text{Gain}(\bar{\varepsilon})$ .

$$u(\bar{Y} + \bar{\varepsilon}) - u(\bar{Y}) \leq \frac{\beta}{1-\beta} \left[ \frac{1}{2} |u''(\bar{Y})| \text{var}(\varepsilon) \right]$$

Note, this is always true for  $\beta \approx 1$ .

What if this isn't satisfied? Is partial insurance still possible?

## Partial Insurance

$$\text{Gain}(\varepsilon_i) = U(\bar{Y} + \varepsilon_i) - U[\bar{Y} + \varepsilon_i - P(\varepsilon_i)] \quad \} \text{ Not as big}$$

$$\text{Cost} = \frac{\beta}{1-\beta} \{ EU[\bar{Y} + \varepsilon - P(\varepsilon)] - EU(\bar{Y} + \varepsilon) \}$$

## Planner's Lagrangian

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^N \pi(\varepsilon_i) U(\bar{Y} + \varepsilon_i - P(\varepsilon_i)) \\ & + \sum_{i=1}^N \lambda(\varepsilon_i) \left\{ \frac{\beta}{1-\beta} \sum_{j=1}^N \pi(\varepsilon_j) [U(\bar{Y} + \varepsilon_j - P(\varepsilon_j)) - U(\bar{Y} + \varepsilon_i)] \right. \\ & \quad \left. - [U(\bar{Y} + \varepsilon_i) - U(\bar{Y} + \varepsilon_i - P(\varepsilon_i))] \right\} \\ & + M \sum_{i=1}^N \pi(\varepsilon_i) P(\varepsilon_i) \end{aligned}$$

FOCs (differentiate w.r.t.  $P(\varepsilon_i)$ ,  $i = 1, 2, \dots, N$ )

$$1.) [\pi(\varepsilon) + \lambda(\varepsilon) + \frac{\beta}{1-\beta} \pi(\varepsilon) \sum_{j=1}^N \lambda(\varepsilon_j)] U'[\mathbf{c}(\varepsilon)] = M \pi(\varepsilon)$$

$$2.) \lambda(\varepsilon) \cdot PC(\varepsilon) = 0 \quad \} \text{ complementary slackness}$$

↑  
Participation Constraint

## 2 cases

1.) Slack PC ( $\lambda = 0$ )

$$\Rightarrow u'[c(\varepsilon)] = \frac{1}{1 + \beta \sum_{i=1}^n \lambda(\varepsilon_i)}$$

$\Rightarrow$  Consumption is state independent

$$\Rightarrow P(\varepsilon) = P_0 + \varepsilon, \quad c(\varepsilon) = \bar{Y} - P_0$$

( $P_0 > 0$  by zero profit constraint)

2.) PC binds ( $\lambda > 0$ )

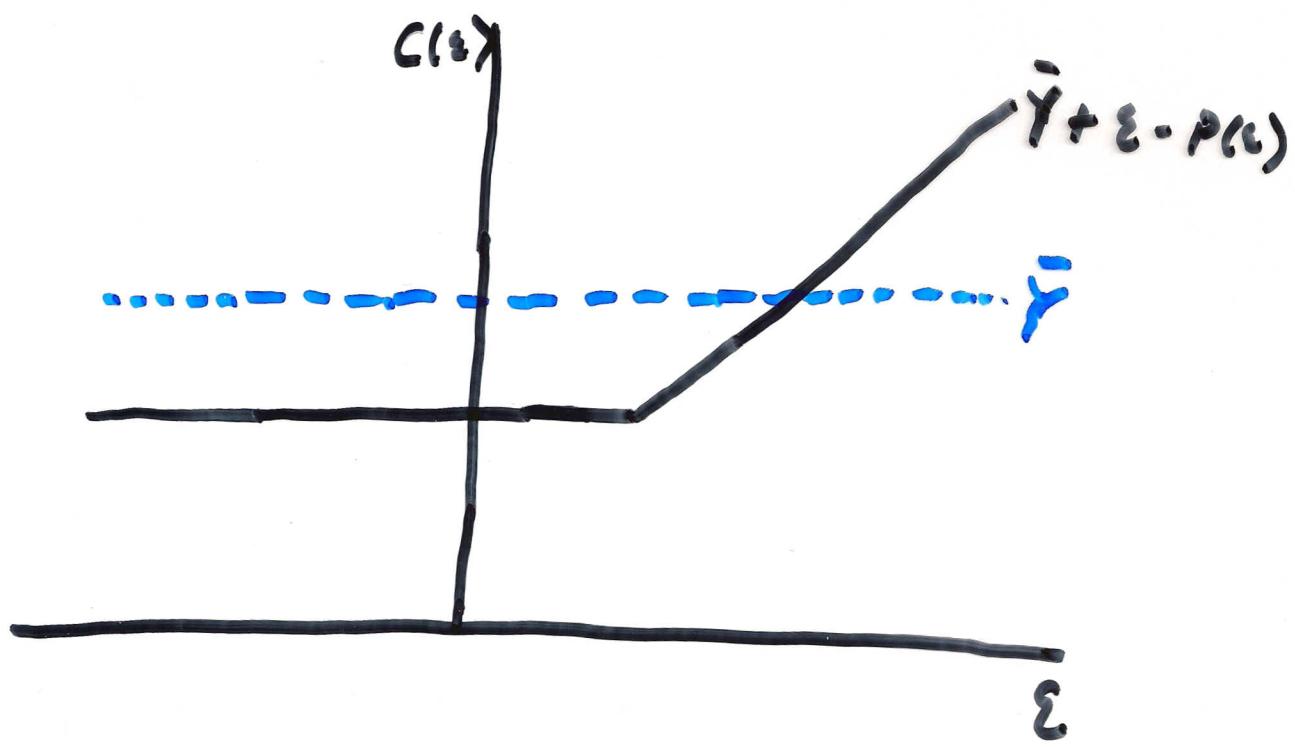
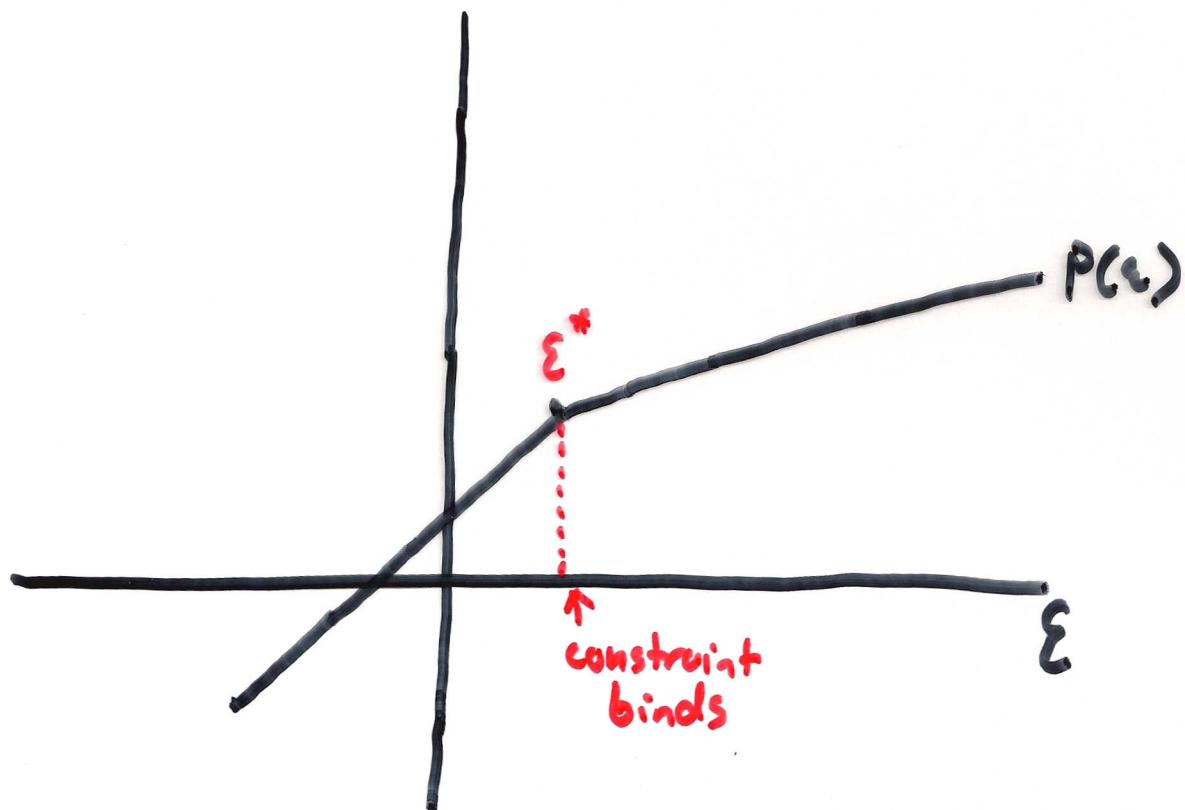
In this case, the constraint defines an implicit function,  $P(\varepsilon)$ .

$$\frac{dP}{d\varepsilon} = \frac{u'[\bar{Y} + \varepsilon - P(\varepsilon)] - u'(\bar{Y} + \varepsilon)}{u'[\bar{Y} + \varepsilon - P(\varepsilon)]} < 1$$

Since  
 $P(\varepsilon) > 0$   
and  $u$   
is concave

Inability to commit implies country must get some of the upside. The cost is that its consumption is lower than  $\bar{Y}$  in the insured states.

## Graphically



## Caveats

- 1.) Saving. If the country can save, then these savings can serve as collateral and enhance insurability (especially if they are deposited in foreign banks!). If  $\beta(1+r) = 1$ , then the country gradually saves enough to eventually obtain full insurance.
- 2.) Bulow-Rogoff (1989). To be sustainable, lenders must be committed and coordinated. That is, being cut-off from the intl. capital market means you can't lend/deposit either. Otherwise, the above contract can be broken by the following strategy - Default at  $\bar{\epsilon}$ , and then invest  $P(\bar{\epsilon})$  in some other bank. Then, sign a new collateralized insurance contract with another lender. You avoid paying  $P(\bar{\epsilon})$  + get to consume the interest!
- 3.) Renegotiation [sub-game perfect vs. Renegotiation Proofness]

Now consider a symmetric treatment of borrowers and lenders (2-sided lack of commitment).

Assume a large number of countries, who want to ensure each other

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s^j)$$

$$Y_t^j = \bar{Y} + \underbrace{\varepsilon_t^j}_{\text{idiosyncratic}} + \omega_t \quad \begin{matrix} \varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}] \\ \omega \in [\underline{\omega}, \bar{\omega}] \end{matrix}$$

$\Downarrow$

$$\sum_j \varepsilon_t^j = 0$$

### Arrow-Debreu

$$C_t^j = \bar{Y} + \omega_t \quad \forall j, t$$

Can this be supported without commitment?

## 2 conditions

- 1.) If country  $j$  defaults, it is permanently shut-out.
- 2.) Other countries also lose their reputations for repaying  $j$ . [This prevents  $j$  from buying collateralized insurance contracts in good states, as in Bulow-Rogoff].

$$\text{Gain}(\varepsilon_+^j, w_+) = U(\bar{Y} + \varepsilon_+^j + w_+) - U(\bar{Y} + w_+)$$

$$\text{Cost} = f_{\beta}^3 [EU(\bar{Y} + w) - EU(\bar{Y} + \varepsilon^j + w)]$$

Sustainable if,  $\text{Gain}(\bar{\varepsilon}, \underline{w}) \leq \text{Cost}$