SIMON FRASER UNIVERSITY
Department of Economics

Econ 842 | Prof. Kasa
International Monetary Economics | Spring 2011

PROBLEM SET 3
(Due April 8)

1. Consider a two-country, one-period model of international risk sharing. Home and Foreign agents have identical utility functions $u(C)$ $[u(C^*)]$. Home's endowment is given by $Y = \bar{Y} + \epsilon$, while Foreign's is given by $Y^* = \bar{Y} - \epsilon$, where ϵ is a zero-mean random shock that is symmetrically distributed around 0 on the interval $[-\bar{\epsilon}, \bar{\epsilon}]$. Assume Home and Foreign agents can write insurance contracts prior to the realization of the relative output shock, which specify a payment by Home to Foreign of $P(\epsilon)$ $[= -P^*(\epsilon)]$.

   (a) Assuming agents can commit, what is the optimal contract? Although the answer should be obvious, formally derive the optimal contract by writing down a Pareto problem, and use the first-order conditions to characterize the contract. Assume equal Pareto weights.

   (b) Now assume that neither agent can commit to the contract. However, assume that if either tries to renege foreign creditors can seize a fraction η of its output. Given this assumption, what are the relevant Participation Constraints?

   (c) Write down the incentive constrained Pareto problem, and use it to characterize the constrained efficient contract. Show that there is an interval $[-e, e]$ such that $C = C^*$ for $\epsilon \in [-e, e]$. Solve for e as a function of η.

   (d) Characterize $C(\epsilon)$ and $C^*(\epsilon)$ when ϵ is outside the interval $[-e, e]$. Use a graph to illustrate your answer.

2. Consider a symmetric two-country, one-good world in which output fluctuations reflect fluctuations in productivity and labor input (there is no capital). The representative agent in country-i has preferences

$$u(c_{it}, 1 - n_{it}) = \log c_{it} + \gamma \log(1 - n_{it})$$

   where c_{it} and n_{it} are consumption and labor in period-t. Output in each country is produced with the linear technology $y_{it} = z_{it} n_{it}$. Each country's productivity shock is i.i.d. Assume that labor is immobile between countries, and that financial markets are complete.

   (a) Write down the optimization problem that characterizes a Pareto optimum. What is the aggregate resource constraint? Derive the first-order conditions.(Hint: Since there is no capital, and the productivity shocks are i.i.d., the problem is essentially static, so it is sufficient to consider only one period).

   (b) For simplicity, assume the Pareto weights are equal. Solve for the Pareto optimal (and competitive equilibrium) allocations of consumption and labor in both countries.

   (c) Explain why output in country 1 depends on the productivity shock in country 2, even though labor is internationally immobile.