Topics for Today

1.) Risk-Sharing & International Portfolio Diversification

- Open-Economy Arrow-Debreu

International Portfolio Diversification

- So far, we have only studied trade in non-contingent assets (bonds). This ignores recent explosive growth of trade of contingent claims.

Contingent Claims $\Longrightarrow$ Anticipate risks/shocks rather than react to them ex post.

$\Longrightarrow$ Smoothing across states rather than smoothing across dates.

- Asset Market structure has an important influence on Current Account dynamics and exchange rates.

- Net vs. Gross Capital Flows

  Gourinchas & Rey (JPE, 07) - Valuation Effects

<u>Example</u>

With only bonds, a temporary (exogenous) negative output shock leads to a CA deficit (borrowing).

However, with complete contingent claims markets, agents could have completely insulated themselves against output shocks. No change in GNP or consumption. (the decline in GDP exactly offset by an increase in payments on net foreign assets).

Arrow-Debreu

- Even without government-imposed barriers, there are still reasons why countries may not be able to achieve full insurance (e.g., enforcement problems, asymmetric info., aggregate risk).

- We're going to start by ignoring these frictions, and study a small open-economy with access to complete contingent claims markets

Assumptions

1.) Domestic MKts. are complete $\Rightarrow$ Representative Domestic Agent

2.) Only 2 dates

3.) Only 2 possible "states" next period

4.) Domestic Residents can trade state-contingent claims

5.) Endowment Economy. [Current Output is known, future output is state-contingent]

Let $\frac{P(1)}{1+r}$ = world price of a claim to output in state 1 [expressed in terms of date 1 output]

$\frac{P(2)}{1+r}$ = world price of a claim to output in state 2 [in terms of date 1 output]

With this choice of numeraire, notice that:

$$P(1) + P(2) = 1$$

} price of a sure claim to output next period is $1+r$

Also note:

1.) A state contingent claim pays off (one unit) if and only if the state it is contingent on is realized next period. Otherwise it pays nothing.

2.) Selling a contingent requires you to deliver state contingent goods. Reneging is not allowed.

3.) Bonds are redundant assets in this economy. Can obtain the same results ~~by~~ with state contingent claims.

4.) Risky assets can be interpreted as bundles of state-contingent claims.

- Let $B_2(s)$ = Domestic residents' net purchase of state-s contingent claims (at date 1)

Budget Constraints

1.) $C_1 + \frac{P(1)}{1+r} B_2(1) + \frac{P(2)}{1+r} B_2(2) = Y_1$ ↗ Known

2.) $C_2(s) = Y_2(s) + B_2(s) \qquad s = 1, 2$

Combine,

$$\boxed{C_1 + \frac{P(1)C_2(1) + P(2)C_2(2)}{1+r} = Y_1 + \frac{P(1)Y_2(1) + P(2)Y_2(2)}{1+r}}$$

Optimization Problem

Preferences satisfy Savage axioms (Expected Utility)

- Sub out C_1 using the budget constraint & optimize w.r.t. $B_2(s)$

$$\boxed{U = u\left[Y_1 - \frac{P(1)}{1+r} B_2(1) - \frac{P(2)}{1+r} B_2(s)\right] + \beta \sum_{s=1}^{2} \pi(s)\, u\left[Y_2(s) + B_2(s)\right]}$$

First-Order Conditions

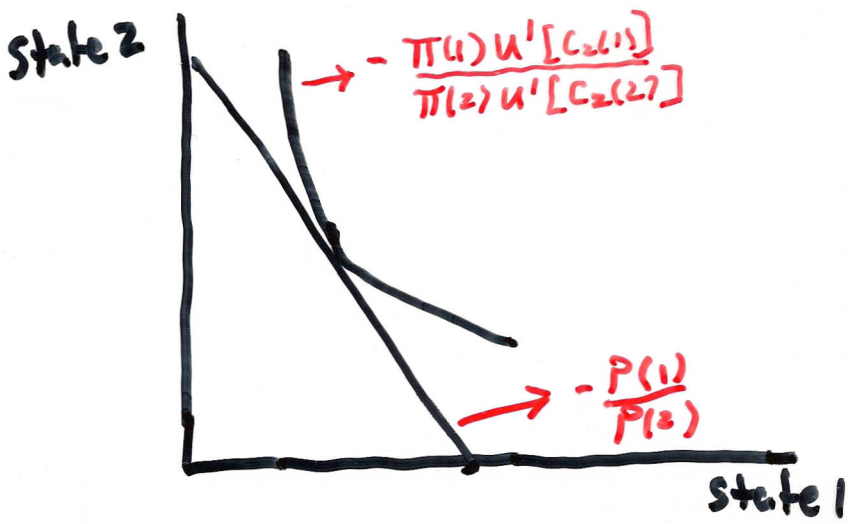
$$\boxed{\frac{P(s)}{1+r} U'(c_1) = \beta \pi(s) U'(c_2(s)) \qquad s=1,2}$$

1.) Add and use fact that $P(1)+P(2)=1$

$$U'(c_1) = \beta (1+r) E_1 U'(c_2)$$ } Standard Euler eq.

2.) Take ratios,

$$\boxed{\frac{P(1)}{P(2)} = \frac{\pi(1) U'[c_2(1)]}{\pi(2) U'[c_2(2)]}}$$ } MRT = MRS



Note, if $\frac{P(1)}{P(2)} = \frac{\pi(1)}{\pi(2)} \Longrightarrow c_2(1) = c_2(2)$

$\Longrightarrow$ Full Insurance (complete smoothing across states)

Log differentiate the MRT = MRS condition,

$$d \log\left[\frac{p(1)}{p(2)}\right] = \frac{u''[c(1)]}{u'[c(1)]} d\,c(1) - \frac{u''[c(2)]}{u'[c(2)]} d\,c(2)$$

$$= \frac{c(1)u''[c(1)]}{u'[c(1)]} d \log c(1) - \frac{c(2)u''[c(2)]}{u'[c(2)]} d \log c(2)$$

Define $\rho = \dfrac{c\,u''(c)}{u'(c)}$

Then,

$$d \log\left[\frac{c(2)}{c(1)}\right] = \frac{1}{\rho} d \log\left[\frac{p(1)}{p(2)}\right]$$

elast. of substitution between state contingent consumption

As $\rho \downarrow$ agent becomes indifferent to consumption across states [Indiff. Curve becomes linear], so agent will concentrate consumption in any state such that $\pi(s) > p(s)$

Equil. with $\rho = 0 \Rightarrow \pi(s) = p(s)$

Risk-Sharing and the Current Account

Assume log utility, $u(c) = \log(c)$

Preferences

$$\log(c_1) + \beta\{\pi(1)\log[c_2(1)] + \pi(2)\log[c_2(2)]\}$$

Wealth

$$W = Y_1 + \frac{P(1)Y_2(1) + P(2)Y_2(2)}{1+r}$$

Basic fact from log util., $c_1 = \frac{1}{1+\beta}W$

$$\frac{P(s)}{1+r}c_2(s) = \frac{\beta}{1+\beta}\pi(s)W$$

Therefore,

$$\boxed{CA_1 = Y_1 - c_1 = \frac{\beta}{1+\beta}Y_1 - \frac{1}{1+\beta}\left[\frac{P(1)}{1+r}Y_2(1) + \frac{P(2)}{1+r}Y_2(2)\right]}$$

Earlier we showed

$r^A > r \implies$ Home autarky price of current consumption is relatively high

$\implies$ Home "imports" current consumption

$\implies$ Home borrows

$\implies$ CA deficit

How can we extend this intuition to a setting of uncertainty and state-contingent claims?

The analysis is trickier now, since there are 3 goods (current consumption + future state-contingent consump)

First, note that if $\beta = \frac{1}{1+r}$ then

1.) $Y_1 = P(1)\,Y_2(1) + P(2)\,Y_2(2) \Rightarrow CA_1 = 0$

2.) $Y_1 < P(1)\,Y_2(1) + P(2)\,Y_2(2) \Rightarrow CA_1 < 0$

This is similar to the non-contingent case

Remember, the CA balance summarizes net resource transfers over time between countries. With state-contingent claims trading, these net transfers can interact in complex ways with trading across states. For example, there can be large differences between a country's net and gross capital flows. Loosely speaking, net flows reflect intertemporal consumption smoothing and gross flows reflect portfolio diversification.

To understand the interaction between net and gross capital flows we must define 3 autarky shadow prices:

(A.) No trade in any financial markets $\Rightarrow c_1 = Y_1$

$c_2(s) = Y_2(s)$

1.) Autarky interest rate

Euler eq. $\Rightarrow$ $$1 + r^A = \frac{u'(Y_1)}{\beta \sum \pi(s) u'[Y_2(s)]}$$

2.) Autarky AD prices, (with log utility)

$$\frac{P^A(s)}{1+r^A} = \frac{\pi(s)\beta Y_1}{Y_2(s)}$$

Write the CA eq. as follows,

$$CA_1 = \frac{1}{1+\beta}\left[\pi(1)\beta Y_1 - \frac{P(1)Y_2(1)}{1+r} + \pi(2)\beta Y_1 - \frac{P(2)Y_2(2)}{1+r}\right]$$

Using the above prices,

$$\boxed{CA_1 = \frac{Y_2(1)}{1+\beta}\left[\frac{P(1)^A}{1+r^A} - \frac{P(1)}{1+r}\right] + \frac{Y_2(2)}{1+\beta}\left[\frac{P(2)^A}{1+r^A} - \frac{P(2)}{1+r}\right]}$$

$\Rightarrow$ If autarky asset prices are high, country is a net importer of assets and runs a CA surplus ($CA > 0$).