SIMON FRASER UNIVERSITY
Department of Economics

Econ 842 — Prof. Kasa
International Monetary Economics — Spring 2011

PROBLEM SET 1 - CURRENT ACCOUNT DYNAMICS
(Solutions)

1. (15 points). Consider a 2-period model of a small open production economy. Assume preferences are

$$\ln(C_1) + \ln(C_2)$$

where ln denotes the natural logarithm. Assume that the initial stock of capital is 100 (i.e., $K_1 = 100$), and the production function in both periods is

$$Q = \sqrt{K}$$

For simplicity, assume that capital completely depreciates during the period, so that $\delta = 1$. Finally, assume the world interest rate is constant at 10% ($r^* = .10$), and the economy's initial net foreign assets are zero (i.e., $B_0^* = 0$).

(a) Compute the firm's optimal investment during period 1, and its resulting period 2 profits, Π_2.

$$MPK = r + \delta \quad \Rightarrow \quad \frac{1}{2}K_2^{-1/2} = 1 + r \quad \Rightarrow \quad K_2 = .2066$$

$$\Pi_2 = F(K_2) - (r+\delta)K_2 = \sqrt{.2066} - (1.1)(.2066) = .2272$$

(b) Solve for the household's optimal consumption in periods 1 and 2.

The Euler equation is

$$\frac{1}{C_1} = \frac{1+r}{C_2}$$

The budget constraint is

$$\begin{aligned} C_1 + \frac{C_2}{1+r} &= W_0 + \Pi_1 + \frac{\Pi_2}{1+r} \\ &= (1+r)K_1 + \{F(K_1) - (1+r)K_1\} + \left\{\frac{F(K_2) - (1+r)K_2}{1+r}\right\} \\ &= Q_1 + \frac{F(K_2)}{1+r} - K_2 \end{aligned}$$

Combining the Euler equation with the budget constraint, along with the answer to part (a), gives

$$\begin{aligned} C_1 &= 5.1033 \\ C_2 &= (1.1)C_1 = 5.614 \end{aligned}$$

(c) Using the fact that $CA_1 = S_1 - I_1$, compute the first period current account balance.

Since $B_0 = 0$, the first-period current account is just

$$CA_1 = S_1 - I_1 = Q_1 - C_1 - I_1 = 10 - 5.1033 - .2066 = 4.69$$

2. (15 points). Consider a 2-period world economy consisting of two countries. Each has preferences

$$U(C_1, C_2) = \ln(C_1) + \ln(C_2)$$

The Home country has endowments $Q_1 = 1$ and $Q_2 = 2$. The Foreign country has endowments $Q_1^* = 2$ and $Q_2^* = 1.3$. Both countries have open capital markets, and both begin with zero net foreign assets.

(a) Compute the equilibrium world interest rate. (Hint: Equilibrium requires $S(r) + S^*(r) = 0$, where $S(r)$ and $S^*(r)$ are the Home and Foreign saving functions, e.g., $S(r) = Q_1 - C_1(r)$).

The savings function in both countries is

$$S(r) = \frac{1}{2}\left(Q_1 - \frac{Q_2}{1+r}\right)$$

In equilibrium, $S(r) + S^*(r) = 0$. This determines the market-clearing interest rate as follows

$$S + S^* = \frac{1}{2}\left[(Q_1 + Q_1^*) - \frac{Q_2 + Q_2^*}{1+r}\right] = 0 \qquad \Rightarrow \qquad 1 + r = \frac{Q_2 + Q_2^*}{Q_1 + Q_1^*} = \frac{3.3}{3} = 1.1$$

(b) Given this interest rate, what are the equilibrium values of Home consumption, C_1 and C_2. Use the above utility function to then compute Home utility.

$$\begin{aligned} C_1 &= \frac{1}{2}\left(1 + \frac{2}{1.1}\right) = 1.409 \\ C_2 &= (1.1)C_1 = 1.5499 \\ U &= \ln(1.409) + \ln(1.5499) = .781 \end{aligned}$$

(c) Now suppose the Foreign country experiences a higher growth rate. In particular, suppose $Q_2^* = 2.5$, with all other endowments remaining the same. What is the new world interest rate? What is Home utility now? Is Foreign growth good or bad for the Home country? Explain.

Now the interest rate is

$$1 + r = \frac{Q_2 + Q_2^*}{Q_1 + Q_1^*} = \frac{4.5}{3} = 1.5$$

and the new values of C_1, C_2 and U are

$$\begin{aligned} C_1 &= \frac{1}{2}\left(1 + \frac{2}{1.5}\right) = 1.167 \\ C_2 &= (1.1)C_1 = 1.75 \\ U &= \ln(1.167) + \ln(1.75) = .714 \end{aligned}$$

Utility is now lower. Foreign growth is bad for Home because it raises the world interest rate, which makes Home worse off since it was initially a borrower (i.e., an adverse terms of trade effect occurs).

3. (25 points). This question examines the implications of a simple modification of the linear-quadratic current account model discussed in class. To set the stage, consider a one-good/one-asset world consisting of a large number of identical countries, each inhabited by a representative agent with preferences

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\} \tag{1}$$

where $u(C_s) = -(a - C_s)^2$. Residents of each country receive a stochastic endowment sequence, $\{Y_s\}$, of the single good. Endowment innovations are independent across countries, giving rise to potential risk-sharing gains. However, assume the only mechanism available for doing this is via ex-post trade in one-period riskless bonds, B_t.

Consider first the case of a single country facing given world market conditions. In particular, suppose the world interest rate is constant, and given by r. Hence, the agent's budget constraint is

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s \tag{2}$$

where B_t is the agent's initial stock of foreign assets. (Note, this constraint must hold with probability one).

(a) Assuming $\beta(1+r) = 1$, write down the agent's Euler equation for consumption.

(b) Substitute this into the agent's budget constraint and derive the agent's consumption function (in terms of the path of expected future endowments). (Note: since the budget constraint holds with probability one, it also holds in expectation).

(c) Using the consumption function in part (b), derive an expression for the current account. Interpret this expression (i.e., explain under what circumstances the country will run current account surpluses or current account deficits).

(a) *The agent's Euler equation is*

$$U'(C_t) = \beta E_t[(1+r)U'(C_{t+1})]$$

Using the facts that $U'(C_t) = a - C_t$ and $\beta(1+r) = 1$, this simplifies to

$$C_t = E_t C_{t+1}$$

That is, consumption follows a martingale.

(b) *Taking expectations of both sides of the intertemporal budget constraint, and using the law of iterated expectations gives*

$$C_t = \left(\frac{r}{1+r} \right) \left\{ (1+r)B_t + E_t \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j Y_{t+j} \right\}$$

That is, the agent consumes the annuity value of his human and financial wealth.

(c) *From the national income accounting identity (using the assumptions that $I \equiv 0$ and $G \equiv 0$),*

$$\begin{aligned} CA_t &= B_{t+1} - B_t \\ &= rB_t + Y_t - C_t \end{aligned}$$

Substituting the answer from part (b),

$$\begin{aligned} CA_t &= Y_t - \left(\frac{r}{1+r}\right) E_t \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j Y_{t+j} \\ &= Y_t - \tilde{Y}_t \end{aligned}$$

That is, the country runs a current account surplus when income is high relative to its annuity/permanent value.

Now let's modify the above model by assuming that agents have preferences of the following recursive form:

$$U_t = u(C_s) + \beta \frac{2}{\sigma} \log E_t[\exp(\frac{\sigma}{2} U_{t+1})] \tag{3}$$

where $\sigma \leq 0$ is a parameter related to risk aversion, and $u(C_s)$ continues to be the quadratic expression $u(C_s) = -(a - C_s)^2$. (Note: this is an example of a class of non-state separable preferences popularized by Epstein and Zin (*Econometrica*, 1989), in their work on asset pricing). For analytical convenience, suppose that endowments follow the process, $Y_t = \bar{Y} + \varepsilon_t$, where $\varepsilon_t \sim i.i.d.\ N(0, 1)$. Also, define the change of variable $\tilde{C}_s = a - C_s$, and the adjusted wealth variable

$$W_t = R \cdot B_t + Y_t + \frac{\bar{Y} - Ra}{r} \tag{4}$$

where $R = (1 + r)$. Using these definitions, we can restate the agent's problem as

$$U_t = \arg\max_{\tilde{C}} \left\{ -\tilde{C}_t^2 + \beta \frac{2}{\sigma} \log E_t \left[\exp\left(\frac{\sigma}{2} U_{t+1}\right)\right]\right\} \tag{5}$$

subject to $W_{t+1} = R(W_t + \tilde{C}_t) + \varepsilon_{t+1}$. Conjecture that $U_{t+1} = -PW_{t+1} - \kappa$, where P and κ are unknown (positive) constants to be determined (via standard fixed point procedures). Then it can be verified that

$$\tilde{C}_t = -FW_t \qquad \text{where} \qquad F = \frac{R \cdot D(P)}{1 + R \cdot D(P)} \tag{6}$$

and $D(P) = P/(1 + \sigma P)$. Solving the Bellman/Riccati fixed point equation yields $P = r/(\sigma + R)$. (Note: you do not need to derive these results, unless of course you like doing algebra). Finally, using these results, we get the following laws of motion for (adjusted) wealth and the current account

$$W_t = R(1 - F)W_{t-1} + \varepsilon_t \tag{7}$$

$$CA_t = \left(\frac{r}{R} - F\right) W_t + \frac{1}{R}\left(Y_t - \bar{Y}\right) \tag{8}$$

Note that in the standard Linear-Quadratic/Certainty-Equivalence case, where $\sigma = 0$, we get the usual result that consumption and wealth follow random walks, and the current account is independent of W_t.

(d) Briefly describe how adjusted wealth and the current account behave when $\sigma < 0$. Explain the intuition. Relate your discussion to the literature on precautionary saving.

Now let's take advantage of an alternative interpretation of the above preferences, using results from Hansen and Sargent's recent monograph entitled *Robustness.* They show that

$$\frac{2}{\sigma}\log E_t\left[\exp\left(-\frac{\sigma}{2}PW_{t+1}\right)\right] = -R^2(W_t+\tilde{C}_t)^2\left(\frac{\theta P}{\theta - P}\right) = \arg\min_{z_{t+1}}[\theta z_{t+1}^2 - R^2(W_t+\tilde{C}_t+z_{t+1})^2 P]$$

where $\theta = -\sigma^{-1}$. Using this, we can recast the agent's problem as a (deterministic) dynamic zero-sum game, with the associated Bellman-Isaacs equation

$$-W_t^2 P = \max_{\tilde{C}_t}\min_{z_{t+1}}[-\tilde{C}_t^2 + \beta\theta z_{t+1}^2 - \beta W_{t+1}^2 P] \tag{9}$$

subject to $W_{t+1} = R(W_t + \tilde{C}_t) + z_{t+1}$. The idea here is that the agent is uncertain (in the Knightian sense) about the stochastic endowment process, so he wants to devise a 'robust' decision rule. As a mechanism for doing this, he imagines a malicious agent chooses a disturbance process, z_{t+1}, so as to subvert his control efforts. That is, the agent plays a game against himself. The parameter θ determines how much freedom the 'evil agent' has. As $\theta \uparrow \infty$, the evil agent's actions become increasingly costly, and so the solution converges to the standard one. Hence, lowering θ produces a more robust decision rule. (Hansen and Sargent describe procedures for calibrating this parameter).

(e) Describe how this economy would respond to a sudden increase ambiguity or uncertainty. In particular, suppose that initially $W_0 < 0$ (below the long-run steady state), and then suddenly θ decreases. How do wealth and the current account respond? How might this result apply to the aftermath of the Asian crisis? (Hint: Refer to recent developments in real interest rates and global current account imbalances).

(f) Compare and contrast this account of recent global imbalances to the work of Caballero, Farhi, and Gourinchas (*AER* 2008) posted on the course webpage.

(d) *When $\sigma < 0$, the agent's preferences are state-nonseparable and exhibit a preference for the early resolution of uncertainty (Kreps and Porteus (1978)). This induces a form of precautionary saving. Normally, in the time-additive/state-separable case, precautionary saving is induced by $u''' > 0$ (i.e., convex marginal utility). Interestingly, we see that this is no longer necessary when preferences are state-nonseparable. Another interesting difference is that here the marginal propensity to consume out of wealth is the same for both human and financial wealth. In traditional models of precautionary saving, this is not the case (see,* *e.g., Caballero (JME, 1990) and Wang (AER, 2003)). Finally, in traditional (partial equilibrium) models of precautionary saving, agents want to accumulate a 'war chest' when $\beta R = 1$, which means that wealth diverges to infinity with probability one. (See the Ljungqvist-Sargent macro text for more disussion). Here that's not the case. Wealth possesses a nice ergodic limiting distribution (notice, however, that this problem ignores non-negativity constraints, which is important for this result).*

Turning to the current account, we get the standard result that the current account responds positively to temporarily high income realizations. This is the second term in eq. (8). More interestingly, notice that the current account exhibits a 'portfolio balance' term as well. The coefficient on W_t turns out to be $\frac{r}{R}\left(\frac{\sigma}{R+\sigma}\right)$. Hence, as you would expect, the current account is the mechanism by which the country achieves a stationary wealth distribution, i.e., it 'borrows' (or, more accurately, sells assets) internationally when (adjusted) wealth is above its steady state, and lends when wealth is below its steady state. (In some respects, this is similar to early models of current account dynamics which featured an endogenous, wealth-dependent, rate of time preference, e..g, Obstfeld (QJE, 1982). This is a common trick to induce stationary equilibria in small open economy models).

(e) *Now, the really interesting thing is that instead of attributing current account dynamics to preferences (i.e., σ), we can use the results of Hansen and Sargent to instead relate it to the macroeconomic* environment *(i.e., θ). Rather than thinking of agents as becoming suddenly more risk averse, we can think of the environment as suddenly becoming more 'ambiguous'. This offers a potentially interesting explanation of some recent international macroeconomic developments. Suppose that after the Asian crisis, residents of the afflicted countries suddenly believed their environment was more ambiguous. After all, the crisis was more severe than any other recent event, and for many people (including economists!), seemed rather difficult to explain. This could be captured by a sudden drop in θ. If we assume that initially $W < 0$ (a defensible assumption for many emerging markets), then we would conclude that Asian countries would respond to the crisis by saving more. Hence, we get a story of Bernanke's 'savings glut'.*

(f) *Caballero, Farhi, and Gourinchas (AER) develop a model in which the crisis led to an erosian in the ability of Asian economies to* supply *financial assets (i.e., a sudden reduction in their δ parameter led to a reduction in collateral). In contrast, the above story focuses on a potential increase in the demand for financial assets. (Warning: in a general equilibrium context, in can be rather misleading to talk about the 'demand' and 'supply' of financial assets, since shocks that shift one curve often shift the other. For example, this is the case in Caballero et. al. It would be an interesting exercise to work out the full general equilibrium implications of this robustness story! The analysis in Hansen, Sargent, and Tallarini (ReStud, 1999) would provide a good starting point).*

4. (15 points). Using the data on the webpage, and whatever software you want, report plots of the current account, as a fraction of GDP, for the U.S., U.K, Japan, and Canada.

5. (30 points). Pick a country, and following the procedure outlined on pages 90-93 of the Obstfeld-Rogoff text, test the Present-Value Model of the current account (i.e., test the model's implied cross-equation restrictions). Plot the model's predicted current account against the actual current account. Comment on the model's fit. (Note: Be sure to express everything in real terms. Although variables should also be expressed in per capita terms as well, don't worry about that. It shouldn't make much of a difference here).