SIMON FRASER UNIVERSITY
Department of Economics

Econ 842 | Prof. Kasa
International Monetary Economics | Spring 2009

PROBLEM SET 3
(Due March 25)

1. (30 points). Consider a 2-period small open endowment economy facing the exogenous world interest rate r on riskless loans. Date 1 output is Y_1. There are S states of nature on date 2 that differ according to the output realizations $Y_2(s)$. The probability that state s is realized is known to be $\pi(s)$. The representative domestic household maximizes the following expected lifetime utility function:

$$U_1 = C_1 - \frac{a}{2}(C_1)^2 + \beta E_1\left[C_2 - \frac{a}{2}(C_2)^2\right] \qquad a > 0$$

Assume that the rate of time preference equals the interest rate, so that $\beta(1+r) = 1$. When markets are *incomplete* the household faces the sequence of budget constraints

$$\begin{aligned} B_2 &= (1+r)B_1 + Y_1 - C_1 \\ C_2(s) &= (1+r)B_2 + Y_2(s) \qquad s = 1, 2 \cdots S \end{aligned}$$

where B_i denotes net foreign assets at the beginning of period-i. Assume that the parameters are such that the marginal utility of consumption, $1-aC$, is always positive.

(a) Start by temporarily ignoring the nonnegativity constraints $C_2(s) \geq 0$ on date 2 consumption. Compute optimal date 1 consumption, C_1. What are the implied values of $C_2(s)$? What do you think your answer would be with an infinite horizon and output uncertainty in each future period? (Hint: Remember chapter 2!).

(b) Now let's worry about the nonnegativity constraint on $C_2(s)$. Without loss of generality, renumber the date 2 states so that $Y_2(1) = \min_s[Y_2(s)]$. Show that if

$$(1+r)B_1 + Y_1 + \frac{2+r}{1+r}Y_2(1) \geq E_1 Y_2$$

then the C_1 computed in part (a) (for the 2-period case) is still valid. What is the intuition? Suppose the preceding inequality doesn't hold. Show that the optimal date 1 consumption is lower (reflecting a precautionary savings effect) and equals

$$C_1 = (1+r)B_1 + Y_1 + \frac{Y_2(1)}{1+r}$$

(Hint: Apply Kuhn-Tucker). What is the intuition here? Does the usual Euler equation hold in this case?

(c) Now assume the household has access to *complete* Arrow-Debreu markets, with $p(s)$ being the exogenous state s Arrow-Debreu contingent claims price for state s. Assume these prices are actuarial fair, so that $p(s) = \pi(s)$. Compute the optimal values of C_1 and $C_2(s)$ in this case. Why can we ignore nonnegativity constraints in this complete markets case?

2. (30 points). Consider a two-country, one-good world where agents in each country have preferences

$$U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\rho}}{1-\rho}$$

Country-1's endowment is $y_{1t} = 1$ for all t. Country-2's endowment is $y_{2t} = \gamma^t$, where $\gamma > 1$.

(a) Describe the competitive equilibrium with complete markets. (Hint: Consider the Pareto problem).

(b) Now suppose agents cannot commit to their Arrow-Debreu contracts, and can go live under autarky at any time. Derive each agent's participation constraints (for each t).

(c) Does the complete markets allocation in part (a) satisfy the participation constraints? If not, what is the constrained-optimal allocation?

3. (40 points). This question is about the trade balance and the terms of trade in open-economy RBC models. Consider a world consisting of two exchange economies, Country 1 and Country 2. Country 1 receives a stochastic endowment sequence of "apples", $a_t(s^t)$, and Country 2 receives a stochastic endowment of "bananas", $b_t(s^t)$, where the notation s^t represents the fact that endowments depend on the history of states realized up to period-t. Residents of both countries have the same preferences

$$U(a, b) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t)[a_t(s^t)^{1-\rho} + b_t(s^t)^{1-\rho}]/(1-\rho)$$

where $\pi(s^t)$ represents the probability of history s^t (so that this is just expected utility).

(a) Compute the Pareto optimal allocation, and describe the supporting prices.

(b) Let q be a country's terms of trade, defined as the the relative price of its imports (so that an decrease in q represents a terms of trade improvement). Compute q for country 1.

(c) Derive an expression for country 1's trade balance, $nx_{1,t} = a_t - q_t b_t$.

(d) What is the relationship between nx_1/y_1 and q, where y_1 is country 1's GDP? What is the relationship between nx_1/y_1 and y_1? Are these consistent with the data?