Econ 305   Prof. Kasa
Intermediate Macroeconomic Theory   Spring 2009

PROBLEM SET 1
(Due January 30)

1. Suppose household preferences are described by the utility function

$$U(C, \ell) = \ln C + \alpha \ln \ell$$

   where C stands for consumption and ℓ stands for leisure. Assuming the market (real) wage is w and the total amount of time available is h, derive expressions for the household's consumption and labor supply decisions as a function of w and h. Interpret the solution.

   Next, suppose a competitive firm's production technology takes the following Cobb-Douglas form

$$Y = zK^\theta N^{1-\theta}$$

   where K stands for capital inputs (assumed to be fixed), N stands for labor inputs, and z is an index of total factor productivity (assumed to be fixed). Derive an expression of the firm's labor demand, as a function of w, K, and z.

2. This problem asks you to put together labor supply and labor demand in order to compute a competitive equilibrium. Now suppose preferences are described by

$$U(C, \ell) = \ln C + \beta \ell$$

   where $\beta > 0$ is a fixed parameter. Continue to assume technology is Cobb-Douglas

$$Y = zK^\theta N^{1-\theta}$$

   Compute the competitive equilibrium values of consumption, employment, output, leisure, and the real wage. What happens to these values when β increases? Interpret your result.

3. Suppose households can earn a higher wage rate for working "overtime". That is, for the first q hours the household works, it receives a real wage of w_1, and for hours worked more than q it receives w_2, where $w_2 > w_1$. For simplicity, assume the household pays no taxes and receives no non-labor income.

   (a) Draw the household's budget constraint, and depict its optimal choice of consumption and leisure.

(b) Show that the household would never work q hours, or anything very close to q hours. Explain this intuitively.

(c) Show what happens if the overtime wage, w_2, increases. Explain the results in terms of income and substitution effects. (Hint: Does it matter whether the household was initially working overtime?)

4. Suppose the government imposes a production/sales tax. That is, the firm pays $\tau < 1$ units of consumption goods to the government for each unit of output it produces and sells. Derive the effect of this on the firm's demand for labor (assume a Cobb-Douglas production function if you want).

   Now suppose the government imposes a tax of τ on *profits* (i.e., revenues minus costs). Derive the effects of this tax on the firm's labor demand. Compare and contrast this result to the one you got for the sales tax. Explain.

5. Consider a Cobb-Douglas production function with <u>three</u> inputs: K is capital (e.g., number of machines), L is labor (e.g., number of workers), and H is "human capital" (e.g., number of college degrees among the workers). Normalizing the scale factor A to one gives us:
$$Y = K^{1/3}L^{1/3}H^{1/3}$$

   (a) Derive an expression for the marginal product of labor. How does an increase in the amount of human capital affect the marginal product of labor?

   (b) Derive an expression for the marginal product of human capital. How does an increase in the amount of human capital affect the marginal product of human capital?

   (c) What is the income share paid to labor? What is the income share paid to human capital? In the National Income Accounts of this economy, what share of total income do you think workers would appear to receive? (*Hint*: Consider where the return to human capital shows up).

   (d) An unskilled worker earns the marginal product of labor, whereas a skilled worker earns the marginal product of labor plus the marginal product of human capital. Using the answers to (a) and (b), find the ratio of the skilled wage to the unskilled wage. How does an increase in the amount of human capital affect this ratio? Explain.

   (e) Some people advocate government funding of college scholarships as a way of creating a more egalitarian society. Others argue that scholarships help only those who are able to go to college. Do your answers to the above questions shed light on this debate?