SIMON FRASER UNIVERSITY
Department of Economics

Econ 305   Prof. Kasa
Intermediate Macroeconomic Theory   Spring 2012

PROBLEM SET 1
(Solutions)

1. (10 points). Using your knowledge of National Income Accounting, briefly describe why the current account deficit and the government budget deficit are often described as 'twin deficits'. Can you think of any reason why they might not be twins?

   Let's start with the National Income Accounting identity:

   $$Y = C + I + G + NX$$

   where Y is domestic GDP. If we add net factor payments to both sides we get

   $$GNP = C + I + G + CA$$

   where CA stands for the Current Account balance (ie, Net Exports of goods and services plus net factor payments). Next, subtract $C + T$ from both sides

   $$GNP - C - T = I + (G - T) + CA$$

   Now, $GNP - C - T$ is just total private sector saving (ie., the difference between disposable income and consumption). Let's call it S_p. Also notice that $G - T$ is just the government budget deficit (ie, the difference between what the government spends and its tax revenues). So let's call it $GBD = G - T$. Finally, since CA represents the Current Account <u>surplus</u>, the current account deficit is just $-CA$. Let's call the Current Account deficit CAD. Using all this notation, we can write the National Income Accounting identity as follows:

   $$CAD = GBD + (I - S_p)$$

   In words, the Current Account deficit must equal the sum of the government budget deficit and the difference between private investment and saving. Notice that if I and S_p remain unchanged, changes in GBD <u>must</u> produce one-for-one changes in CAD. In this sense, the Current Account and government budget deficit are 'twins'. If the government borrows more, and the private sector isn't prepared to lend it the money, it must borrow it from foreigners (ie, run a Current Account deficit). In practice, it is often the case that the two deficits do move together, but not always. Depending on the circumstances, there might be reasons why the private sector adjusts its saving in response to the government's saving (after all, the government is just the instrument of the people!) Later (in chpt 8), we'll see that if the government saves less, the private sector might decide to save more, in anticipation of the higher taxes they will need to pay in order to service the larger government deficit. This

offsetting behavior is called 'Ricardian Equivalence'. Another simple reason the two deficits may diverge is if investment changes. Sometimes, in response to an investment boom, a country will decide to borrow from abroad, in order to fund the investment. This will produce a temporary current account deficit, without any necessary change in the government budget. A good example of this when countries discover a natural resource, like oil.

2. (20 points). Suppose household preferences are described by the utility function

$$U(C, \ell) = 2\sqrt{C} + \alpha\ell$$

where C stands for consumption of market goods and ℓ stands for leisure.

(a) Assuming the market (real) wage is w and the total amount of time available is h, derive expressions for the household's consumption and labor supply decisions as a function of w and h. (For simplicity, assume the household has no nonmarket income). Does the income effect ever dominate the substitution effect. How would things change if the household had some nonmarket income? Explain intuitively.

The household's optimality condition is

$$\frac{U_\ell}{U_c} = w$$

Computing the derivatives and substituting in gives you

$$\alpha\sqrt{C} = w \qquad \Rightarrow \qquad C = \left(\frac{w}{\alpha}\right)^2$$

The household's budget constraint can be written

$$C + w \cdot \ell = w \cdot h$$

Substituting in the above expression for C and simplifying gives

$$\frac{w}{\alpha} + \ell = h \qquad \Rightarrow \qquad N^s = h - \ell = \frac{w}{\alpha}$$

Notice a couple of funny things here: (1) Consumption only depends on the wage rate! It doesn't depend on either leisure, or the total time endowment, (2) Labor supply only depends on the wage too, and is also independent of the total time endowment. In other words, if God gave this person an extra 10 hours a day, she would use it all up as leisure. She wouldn't work an extra minute (assuming of course the wage stays the same). Why is this? What's going on here? Notice that preferences here are <u>linear</u> in ℓ. The marginal utility of leisure is constant, not diminishing. (These kinds of preferences are called 'quasi-linear'). As a result, this person/household will consume all extra income (beyond a certain point) in the form of leisure. The income effect on consumption is zero! Geometrically, as the budget constraint shifts out in a parallel fashion, the tangency points all occur at the same level of consumption! Obviously then, without having to do any math, we know that if this person gets extra non-market income, consumption and labor supply will remain unchanged.

(b) Now suppose output, Y, is produced by competitive firms with technology

$$Y = zK^{1/2}N^{1/2}$$

where K denotes (fixed) capital inputs, N denotes labor inputs, and z is an index of total factor productivity. Derive an expression of the firm's labor demand, as a function of w, K, and z.

If the firm is maximizing profits, it will hire labor up to the point where the marginal product of labor equals the (real) wage rate. That is,

$$\frac{1}{2}zK^{1/2}\frac{1}{\sqrt{N}} = w \qquad \Rightarrow \qquad N^d = \frac{z^2K}{4w^2}$$

(c) Using your answers to parts (a) and (b), derive an expression for the market-clearing wage rate. (For simplicity, assume the firm's profits do not accrue to the household, so that the labor supply curve is the same as in part (a)). How does the equilibrium wage change when z increases? How does the equilibrium wage change when α increases? Use a Labor Supply/Labor Demand graph to illustrate these changes.

Equilibrium occurs when labor supply equals labor demand. Using the answers to parts (a) and (b) gives

$$N^s = \frac{w}{\alpha} = \frac{z^2K}{4w^2} = N^d \qquad \Rightarrow \qquad w^3 = \frac{\alpha z^2K}{4} \qquad \Rightarrow \qquad w = \left(\frac{\alpha z^2K}{4}\right)^{1/3}$$

Notice that the equilibrium wage increases when α increases. An increase in α shifts up the labor supply curve. This produces both an increase in the wage and a reduction in equilibrium employment/hours.

3. (20 points). Over the past 100 years or so, the development experience of many countries is described by two facts: (1) The economy's capital/labor ratio rises, and (2) The share of labor and capital in national income are roughly constant. How can this be? What do these two empirical facts suggest about the form of the production function? (Hints: (1) The share of labor is $w \cdot L/Y$ and the share of capital is $R \cdot K/Y$, (2) Assume factor markets are competitive, so factors are paid their marginal products).

On the surface, it would seem that as K/L increases during the development process, we should see an increase in the share of capital in national income. After all, capital is becoming relatively more abundant. However, this ignores the fact that changes in K/L will produce equilibrium changes in wages and capital rental rates. In particular, in competitive markets, factors are paid their marginal products, and because of diminishing returns, when K/L rises it causes the marginal product of capital to fall and the marginal product of labor to rise. It turns out that with a constant returns to scale Cobb-Douglas production function, the wage and rental adjustments exactly offset the changes in K/L, so that factor shares remain unchanged. (Technically, this happens when the elasticity of substitution between capital and labor is exactly one, like it is with a Cobb-Douglas production function). More formally, if $Y = AK^\alpha L^{1-\alpha}$ we have

$$w = \alpha AK^\alpha L^{-\alpha} = \alpha A\left(\frac{K}{L}\right)^\alpha$$

$$R = (1-\alpha)AK^{\alpha-1}L^{1-\alpha} = (1-\alpha)A\left(\frac{L}{K}\right)^{1-\alpha}$$

Now, notice what happens when we divide capital's share by labor's share

$$\frac{Capital Share}{Labor Share} = \frac{R}{w}\frac{K}{L} = \frac{1-\alpha}{\alpha}$$

In other words, the relative price effect __exactly__ offsets the relative quantity effect!

4. (20 points). In the 1930's Keynes famously argued that the best way to stimulate employment was to pay people to dig holes and fill them up again. This question explores the logic behind that advice.

   (a) Suppose households have the following preferences:

   $$U(C, \ell, G) = \sqrt{C+G} + \sqrt{\ell}$$

   where, as before, C is market consumption and ℓ is leisure. The new element here is government purchases, G. Note that the above preferences imply that government-provided goods are perfect substitutes for private consumption. Of course, the government must pay for the goods it buys, and suppose it does so by imposing lump-sum taxes, T, on the household. Since there is only one period, the government budget constraint is just $G = T$. Now the household's budget constraint is

   $$C + w \cdot \ell = w - T = w - G$$

   For simplicity, suppose that the production takes the simple form,

   $$Y = N = 1 - \ell$$

   so that the marginal product of labor (and therefore the wage) is constant and equal to one. Given this information, solve for the household's labor supply curve. Does it depend on G? Why or why not? What happens to output, employment, and consumption when G increases?

   Once again, the household's optimality condition is

   $$\frac{U_\ell}{U_c} = w$$

   Taking the derivatives (using the chain rule) gives

   $$\sqrt{\frac{C+G}{\ell}} = w \qquad \Rightarrow \qquad \frac{C+G}{\ell} = w^2$$

   The budget constraint now is a little different, since the household must pay taxes to finance government spending. It now becomes

   $$C + w \cdot \ell = w \cdot h - T$$

   In this simple one-period model the government budget constraint implies $G = T$. Substituting this along with the household's optimality condition into the budget constraint gives

   $$C + G = w(h - \ell) = wN^s \qquad \Rightarrow \qquad N^s = w\ell \qquad \Rightarrow \qquad N^s = \frac{w}{1+w}h$$

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Hence, the household works a fraction of its total available time, where the fraction is increasing in w. However, notice that labor supply does not depend on G. Neither does labor demand, since the marginal product of labor is constant (and equal to one). Therefore, output and employment do not depend on fiscal policy. This is because government-supplied services are perfect substitutes for private consumption. When the government raises taxes to buy things, the private sector just lowers its consumption one-for-one. There is no adverse wealth effect from higher taxes, since the government is buying things the household would have bought anyway!

(b) Now suppose the government buys useless things (e.g., holes in the ground), so that the household's utility function is as follows:

$$U(C, \ell) = \sqrt{C} + \sqrt{\ell}$$

In other words, government purchases do not provide utility to the household. Does labor supply now depend on G? Why or why not? (Hint: Your answer should refer to income and substitution effects). If G increases, how does output and employment respond? Was Keynes right? (WARNING: This is a neoclassical economy. There is no unemployment! Keynes had in mind a different sort of economy, but this example sheds some light on his thinking).

We just need to repeat the above analysis, with the new utility function. The household's optimality condition becomes

$$\frac{C}{\ell} = w^2$$

Substituting this into the budget constraint now yields

$$N^s = \frac{w}{1+w}h + \frac{G}{w(1+w)}$$

Notice that now when G increases, labor supply <u>increases</u>. Why the difference? Now the government is buying things people don't care about (ie, digging holes). Raising taxes to pay for these things represents a negative wealth effect, which makes people work harder. There is no offsetting substitution effect here since wages are pinned down by the constant marginal product of labor. Therefore, Keynes was right! If your goal is to stimulate employment, then have the government buy useless things! (Of course, in this economy, where there are no distortions, doing this makes people <u>worse off</u>! There is an important difference between output and welfare). The economy Keynes had in mind was subject to monopoly distortions, where output is demand-determined, at least in the short-run. That is, Keynes was more concerned with the demand-side than the supply-side. If output is sub-optimally low, then stimulating aggregate demand is useful. Doing this is more effective when the government buys useless things, since it doesn't crowd out private spending. We'll discuss this more later on.