

## A Simple Model of the Current Account

- Last time we saw that a nation's current acct. balance reflects a discrepancy between its saving and investment:

$$\boxed{CA = S - I}$$

To understand current accts. + capital flows, we therefore need a dynamic model.

- It is useful to think of a small open economy operating in the global capital market exactly like an individual household operating in a (closed) domestic capital market. As with households, the key intuition is consumption-smoothing, and is formalized by Friedman's "Permanent Income Hypothesis".
- Our basic strategy is to begin slowly, with the simpliest possible model, and then add real-world complications gradually, one at a time.

## Assumptions

- 1.) Only 2 periods
- 2.) Small Open Economy (exogenous interest rate)
- 3.) Endowment Economy (No production)
- 4.) 1 good (only intertemporal gains from trade).
- 5.) No Uncertainty / 1 asset
- 6.) Representative Agent (Domestic markets are complete)

## Budget Constraints

$$C_1 + B_1^* - B_0^* = r_0 B_0^* + Q_1 \quad \} \text{ period 1}$$

$$C_2 + B_2^* - B_1^* = r_1 B_1^* + Q_2 \quad \} \text{ period 2}$$

Note, the country has 2 sources of income:  
 1.)  $Q$ : output and 2.)  $rB$ : investment income.  
 Likewise, it has 2 uses for income: 1.)  $C$ : consumption  
 and 2.)  $B_+ - B_{-}$ : saving/asset accumulation.

## "Transversality Condition"

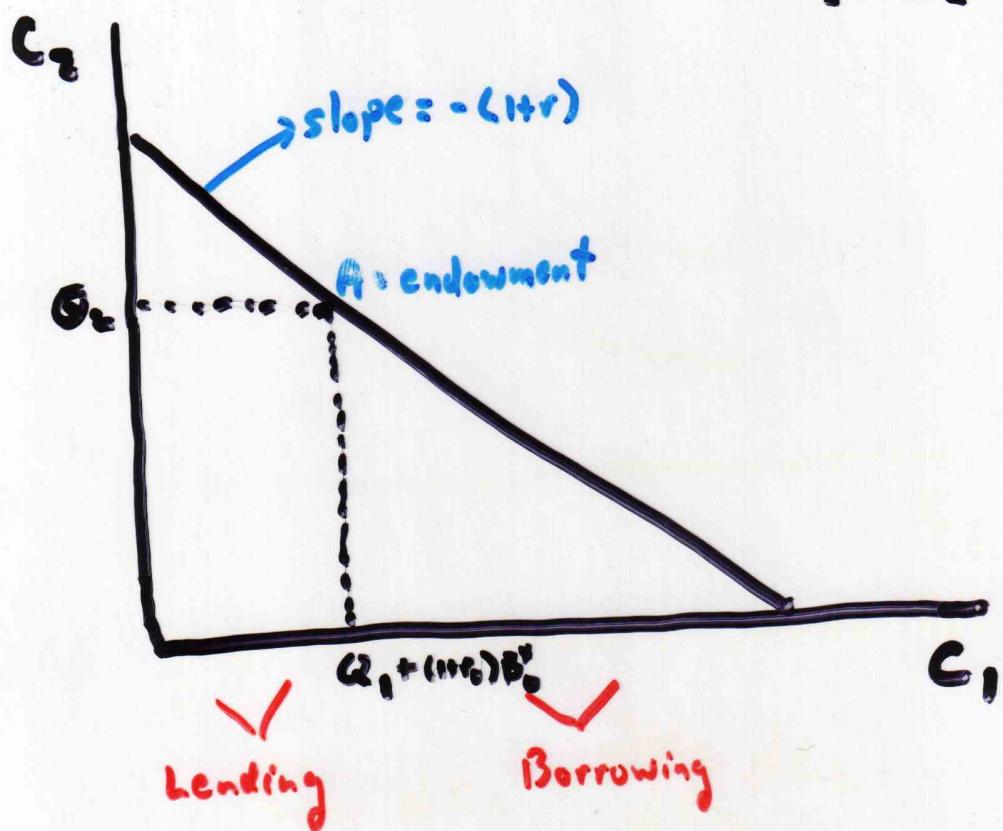
Since the world ends after period 2, a country would never choose  $B_2^* > 0$ . Doing so would be throwing money away! At the same time, it is not possible for  $B_2^* < 0$  (why?). Hence, we have the transversality condition,  $B_2^* = 0$ .

- Imposing the transversality condition, and then substituting period 2's budget constraint into period 1's budget constraint, we get the following inter-temporal budget constraint:

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}$$

We can graph this as follows,

$$C_2 = Q_2 - (1+r) \{ (1+\epsilon) B_0^+ + Q_1 - C_1 \}$$



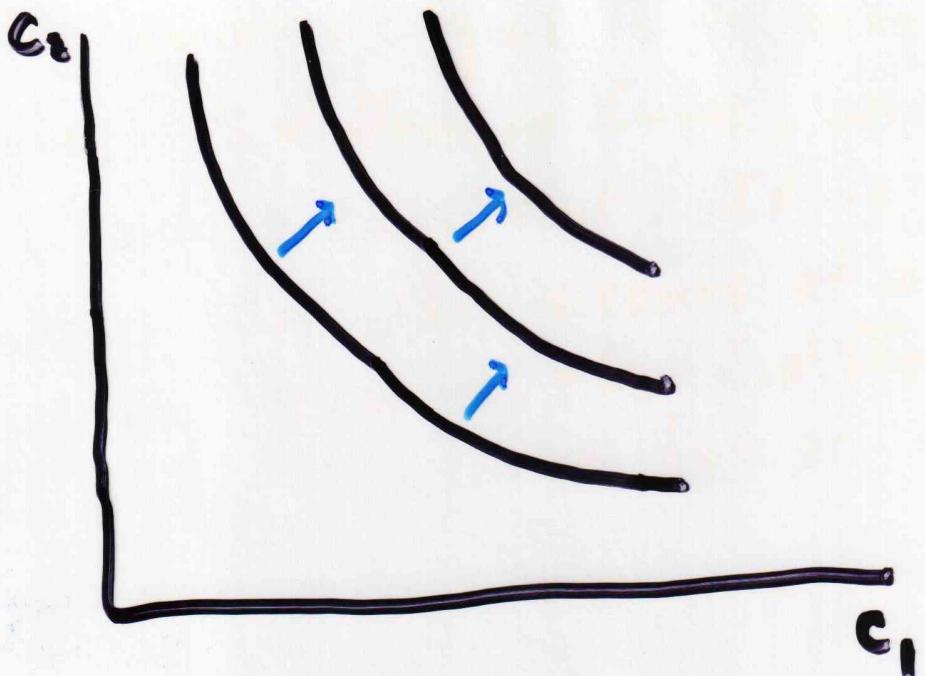
To determine which point the country chooses, we need to consider its preferences

## Preferences

Preferences can be described by a "utility function"

$$U(C_1, C_2)$$

The essential aspects of preferences are contained in its "Indifference Curves".



Why are the indifference curves downward sloping and "convex"?

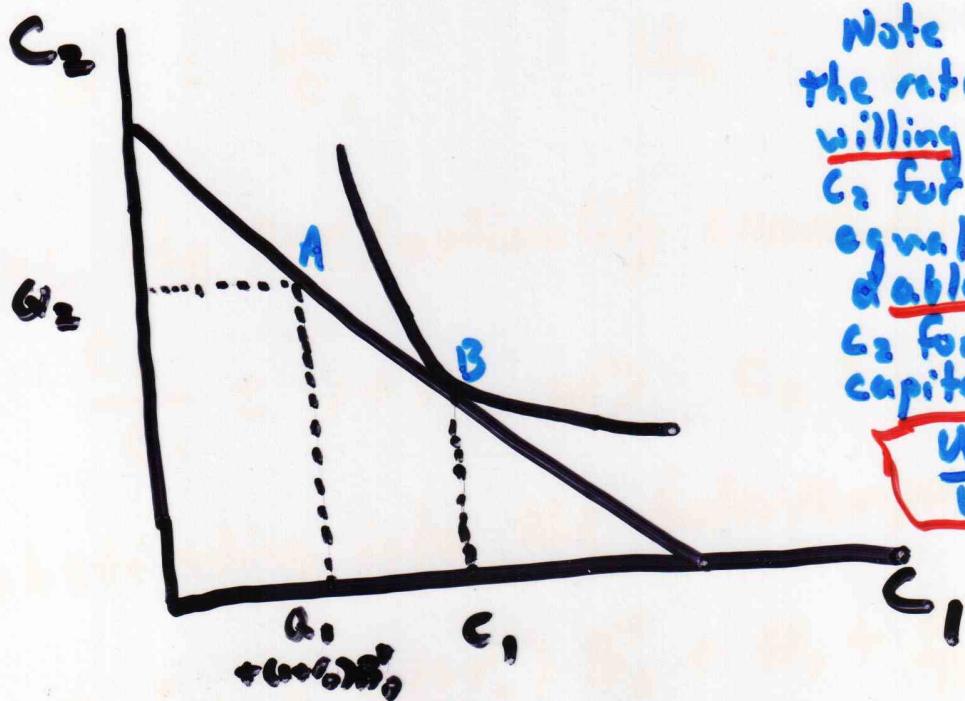
Along an indifference curve we have

$$\begin{aligned} du &= \frac{\partial u}{\partial c_1} dc_1 + \frac{\partial u}{\partial c_2} dc_2 \\ &\therefore u_1 dc_1 + u_2 dc_2 = 0 \end{aligned}$$

$$\Rightarrow \boxed{\frac{dc_2}{dc_1} = -\frac{u_1}{u_2}}$$

The slope of an Indifference Curve,  $-\frac{u_1}{u_2}$ , is called the Marginal Rate of Substitution. It tells you how willing the consumer is to substitute  $C_2$  for  $C_1$ . Convexity of the indifference curves reflects a "diminishing marginal rate of substitution".

The country's goal is to reach the highest Indifference Curve, while staying within its budget constraint. It is geometrically obvious that this occurs at a point of tangency between the Indifference Curve and the budget constraint [at pt. B below] :



Note that in equilibrium the rate the country is willing to substitute  $C_2$  for  $C_1$  is exactly equal to the rate it is able to substitute  $C_2$  for  $C_1$  on the world capital market. That is,

$$\frac{U_1}{U_2} = 1+r$$

Current Acct.  
Deficit

Note that this country runs a Current Acct. deficit because it consumes,  $C_1$ , more than its 1st period income,  $(Q_1 + (1+r)B_0)$ . That is, it chooses to borrow.

## An Algebraic Example

Suppose we have

$$U(c_1, c_2) = \ln c_1 + \ln c_2$$

Notice that in this case,

$$U_1 = \frac{1}{c_1}, \quad U_2 = \frac{1}{c_2}$$

Thus, the FOC/optimality condition is:

$$\frac{c_2}{c_1} = 1+r, \Rightarrow c_2 = (1+r)c_1$$

Substituting into the intertemporal budget constraint

$$c_1 + c_2 = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}$$

$= \bar{Y}$  (present discounted value)  
of wealth

$$\Rightarrow c_1 = k \bar{Y}$$

Therefore,

$$TB_1 = Q_1 - c_1 = \frac{1}{2} [Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r_1}]$$

$$CA_1 = r_1 B_0^* + TB_1 = r_1 B_0^* + \frac{1}{2} [Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r_1}]$$

## Current Account Sustainability

- Popular press commentators often assert that a given current acct. imbalance is "unsustainable". What exactly does this mean? One interpretation is that countries cannot run current acct. deficits forever. Eventually, a current acct. deficit must be matched by future surpluses. Is this true?
- Perhaps surprisingly, our very simple model allows us to answer these questions.

### Finite Horizons

First consider the case of finite horizons, for simplicity, just 2 periods. Also assume for simplicity that the world interest rate,  $r$ , is constant.

By definition, we know

$$(1) \quad CA_1 = r B_0^* + TB_1,$$

$$(2) \quad CA_1 = B_1^* - B_0^*$$

Eliminating CA, from (1) and (2) gives

$$(3) \quad B_1^* = (1+r)B_0^* + TB,$$

For period 2 this gives

$$(4) \quad B_2^* = (1+r)B_1^* + TB_2$$

Eliminating  $B_1^*$  from (3) and (4), and imposing the transversality condition,  $B_2^* = 0$

$$(5) \quad (1+r)B_0^* = -TB_1 - \frac{TB_2}{1+r}$$

Already we have an interesting result:

Unless a country has a positive initial net foreign asset position ( $B_0^* > 0$ ), it is not possible for a country to run perpetual trade deficits. A period of trade deficits must be offset by future trade surpluses. On the other hand, if a country does have a positive initial net foreign asset position, it can run a perpetual TB deficit.

What about the Current Acct. ?

Again by definition,

$$CA_2 = B_2^* - B_1^*$$

Using (2) to eliminate  $B_1^*$ , and imposing  
 $B_2^* = 0$ ,

$$B_0^* = -CA_1 - CA_2$$

Again we have the same result,

A country cannot run perpetual Current Acct. deficits unless it has a positive initial net foreign asset position.

## Infinite Horizons

- The previous results seem to support the popular press dictum that countries cannot run perpetual CA deficits.
- Interestingly, we shall now see that if the economy lasts forever, then it is possible to run perpetual CA deficits, even when the economy begins with zero net foreign assets!

Rewriting (3) gives us,

$$(6) \quad B_0^* = \frac{B_1^*}{1+r} - \frac{T B_1}{1+r}$$

shifting forward one period

$$B_1^* = \frac{B_2^*}{1+r} - \frac{T B_2}{1+r}$$

Sub into (6)

$$B_0^* = \frac{B_1^*}{(1+r)^2} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2}$$

Iterating forward for  $T$  periods,

$$B_0^* = \frac{B_T^*}{(1+r)^T} - \frac{TB_1}{(1+r)} - \frac{TB_2}{(1+r)^2} - \dots - \frac{TB_T}{(1+r)^T}$$

Now the Transversality Condition is more subtle. It must prevent "Ponzi Games", where current debts are paid by issuing new debt. It precludes,

$$\lim_{T \rightarrow \infty} \frac{B_T^*}{(1+r)^T} < 0$$

At the same time, it would not be optimal to have  $\lim_{T \rightarrow \infty} \frac{B_T^*}{(1+r)^T} > 0$ . Hence, the Transversality Condition becomes,

$$\boxed{\lim_{T \rightarrow \infty} \frac{B_T^*}{(1+r)^T} = 0}$$

Note: This does not require a country to "pay off" its debt. It just prevents debt from growing too fast.

Letting  $T \rightarrow \infty$  and imposing the  $TB_C$ ,

$$\tilde{B}_0 = - \sum_{j=1}^{\infty} \frac{TB_j}{(1+r)^j}$$

So again, even with infinite horizons, it is not possible to run perpetual  $TB$  deficits without having a positive initial net foreign asset position.

What about the Current Act.?

Suppose the economy begins with  $B_0^* < 0$ . The no-Ponzi game condition prevents the country from rolling over its entire interest payment each period, so suppose the country pays a fraction,  $\alpha$ , of its interest obligations each period, which requires it to run a  $TB$  surplus

$$TB_t^* = -\alpha r B_{t-1}^*$$

With this debt-servicing plan, the CA is as follows

$$CA_t = r B_{t-1}^* + TB_t$$

$$= r(1-\alpha) B_{t-1}^*$$

And foreign debt evolves as:

$$\bar{B}_t^* = (1+r) B_{t-1}^* + TB_t$$

$$= (1+r - \alpha r) B_{t-1}^*$$

$$\Rightarrow B_t^* = (1+r - \alpha r)^t B_0^*$$

Notice that since  $B_0^* < 0$ :

- 1.) The country runs perpetual CA deficits
- 2.) The country never "pays off" its foreign debt.  $B_t^* < 0 \quad \forall t$

However, the key question is whether the TVC holds! Since  $B_t^* = (1+r-\alpha r)^t B_0^*$ , we have :

$$\frac{B_t^*}{(1+r)^t} = \left[ \frac{1+r(1-\alpha)}{1+r} \right]^t B_0^*$$

Notice that since  $1+r > 1+r(1-\alpha)$  we have

$$\lim_{t \rightarrow \infty} \frac{B_t^*}{(1+r)^t} = 0 \Rightarrow \text{TVC holds!}$$

The final issue is whether it is possible for the country to run the required TB surpluses.

By definition,

$$\begin{aligned} TB_t &= B_t^* - (1+r) B_{t-1}^* \\ &= -\alpha r [1+r(1-\alpha)]^{t-1} B_0^* \end{aligned}$$

This implies the TB grows at rate  $r(1-\alpha)$ . Obviously, the trade surplus cannot exceed GDP! Therefore, for this to be feasible, it must be the case that

$$g > r(1-\alpha)$$

where  $g$  is the economy's growth rate.

This can be expressed as a minimum payment condition:

$$\alpha > \frac{r-g}{r}$$

Notice that the faster the economy grows, the smaller is the required debt service.

Although it might appear as if no debt service at all is required if  $g > r$ , it turns out that  $g > r$  is not consistent with our original small country assumption. Countries that grow faster than the interest rate for long periods eventually become large! Remember,  $r$  is an equilibrium price, and it cannot be lower than all countries' growth rates.)